A STUDY OF SOLVING RECURRENT SEQUENCES

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ABSTRACT

In the process of solving many mathematical and mathematical Olympiad problems, the sample solution is expressed using a recurrent formula. Solving this recurring formula is not always easy. This thesis presents several methods and solutions for solving recurrent sequences of this type.

KEYWORDS: *Recurrent Sequence, Initial Condition, General Term, Coefficients.*

INTRODUCTION

In the Mathematical Olympiad, as well as in mathematical analysis, in many problems of algebra, the solution is found in the form of a recurrent sequence, and it is always relevant and important to solve this sequence and find its general term on the basis of initial conditions. On this basis, the following are methods and solutions for finding the general term of several recurrent sequences.

Problem 1. Find the formula of the general term for the recurrent sequence $u(n+1) = u(n) - u(n-1)$given by the initial conditional $u(1) = a, u(2) = b,\ n > 1$.

Solution:

To solve this recurrent sequence, we assign a value to n, starting from the initial value:

$$if n = 2\ then u(3) = u(2) - u(1) = b - a;$$

$$if n = 3\ then u(4) = u(3) - u(2) = -a;$$

$$if n = 4\ then u(5) = u(4) - u(3) = -b;$$

$$if n = 5\ then u(6) = u(5) - u(4) = -(b - a);$$

$$if n = 6\ \ then u(7) = u(6) - u(5) = a;$$

$$if n = 7\ then u(8) = u(7) - u(6) = b;$$

$$if n = 8\ \ then u(9) = u(8) - u(7) = b - a;$$

It can be seen that the terms of the recurrent sequence are repeated every 7 values. In this case, the general term of a given recurrent sequence can be written as follows:

$$u(n)=\begin{cases}(-1)^{k+1}(b-a), & if n=3k\\ (-1)^{k}a, & if n=3k+1\\ (-1)^{k}b, & if n=3k+2\end{cases}.$$

Problem 2. Find the formula of the general term for the recurrent sequence $u(n+1)=\frac{u(n)}{u(n-1)}$given by the initial conditional $u(1)=a, u(2)=b, n>1$.

Solution:

As above, we give a sequence of values to the sequence:

$$if n=2\ then u(3)=\frac{u(2)}{u(1)}=\frac{b}{a};$$

$$if n=3\ then u(4)=\frac{u(3)}{u(2)}=\frac{1}{a};$$

$$if n=4\ then u(5)=\frac{u(4)}{u(3)}=\frac{1}{b};$$

$$if n=5\ then u(6)=\frac{u(5)}{u(4)}=\frac{a}{b};$$

$$if n=6\ then u(7)=\frac{u(6)}{u(5)}=a;$$

$$if n=7\ then u(8)=\frac{u(7)}{u(6)}=b;$$

$$if n=8\ then u(9)=\frac{u(8)}{u(7)}=\frac{b}{a};$$

It can be seen that the terms of the recurrent sequence are repeated every 7 values. In this case, the general term of a given recurrent sequence can be written as follows:

$$u(n)=\begin{cases}\left(\frac{a}{b}\right)^{(-1)^{k}}, & if n=3k\\ a^{(-1)^{k}}, & if n=3k+1\\ b^{(-1)^{k}}, & if n=3k+2\end{cases}.$$

Problem 3. Find the formula of the general term for the recurrent sequence $u(n)^2=C\cdot u(n-1),\ n\geq 1$given by the initial conditional $u(0)=a$.

Solution:

We determine the regularity by assigning a value to the recurrent sequence:

$$if n=1\ then u(1)^2=C\cdot u(0)=C\cdot a \Rightarrow u(1)=(Ca)^{\frac{1}{2}};$$

$$if n = 2\ then u(2)^2 = C \cdot u(1) = C \cdot (Ca)^{\frac{1}{2}} \Rightarrow u(2) = (C^3 a)^{\frac{1}{4}};$$

$$if n = 3\ then u(3)^2 = C \cdot u(2) = C \cdot (C^3 a)^{\frac{1}{4}} \Rightarrow u(3) = (C^7 a)^{\frac{1}{8}};$$

$$if n = 4\ then u(4)^2 = C \cdot u(3) = C \cdot (C^7 a)^{\frac{1}{8}} \Rightarrow u(4) = (C^{15} a)^{\frac{1}{16}};$$

Therefore, the general term of a given recurrent sequence can be written as follows:

$$u(n) = \left(C^{2^n - 1} a\right)^{\frac{1}{2^n}}.$$

Problem 4. Find the formula of the general term for the recurrent sequence $u(n) = \frac{An+B}{Cn+D} u(n-1), n \geq 1$given by the initial conditional $u(0) = a$.

Solution:

We determine the regularity by assigning a value to the recurrent sequence:

$$if n = 1\ then u(1) = \frac{A+B}{C+D} \cdot u(0) = \frac{A+B}{C+D} \cdot a;$$

$$if n = 2\ then u(2) = \frac{2A+B}{2C+D} \cdot u(1) = \frac{(A+B)(2A+B)}{(C+D)(2C+D)} \cdot a;$$

$$if n = 3\ then u(3) = \frac{3A+B}{3C+D} \cdot u(2) = \frac{(A+B)(2A+B)(3A+B)}{(C+D)(2C+D)(3C+D)} \cdot a;$$

$$if n = 4\ then u(4) = \frac{4A+B}{4C+D} \cdot u(3) = \frac{(A+B)(2A+B)(3A+B)(4A+B)}{(C+D)(2C+D)(3C+D)(4C+D)} \cdot a;$$

Therefore, the general term of a given recurrent sequence can be written as follows:

$$u(n) = \frac{(A+B)(2A+B)\ldots(nA+B)}{(C+D)(2C+D)\ldots(nC+D)} \cdot a = \prod_{k=1}^{n} \frac{kA+B}{kC+D} \cdot a.$$

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