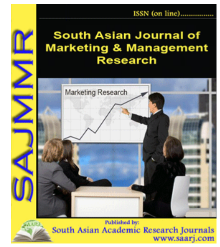
SAJMMR

ISSN: 2249-877X Vol. 11, Issue 8, August 2021, Impact Factor: SJIF 2021= 7.642

South Asian Journal of Marketing & Management Research (SAJMMR)

(Double Blind Refereed & Peer Reviewed International Journal)



DOI: 10.5958/2249-877X.2021.00053.9

VARIOUS WAYS OF SOLVING EXTREMUM PROBLEMS

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ABSTRACT

This article deals with extremum problems solved in various ways. Demonstrated different approaches to problems. The presented different solutions are compared and it is determined that each is distinguished by the naturalness of reasoning, while the other is somewhat artificial. The solutions of extreme problems by different methods are shown and the choice of them contributes to the development of students creative thinking.

KEYWORDS: *Extremum, Segment, Vector, Parallel, Perpendicular, The Method Of Lagrange Multipliers.*

INTRODUCTION

It has already become a tradition to use it in any extreme problem. Meanwhile, for many of these problems, there are other ways of solving algebraic, geometric, which were used for centuries, until differential and integral calculus was invented.

But even after the development of methods of analysis, the techniques of algebra and geometry were not forgotten, and in many cases turned out to be preferable to new methods. It is hardly worth turning a blind eye to this fact and turning to derivatives, even in cases where it is easier to do without them.

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The purpose of the article, admitting various solutions, is usually interesting and instructive. Each of them demonstrates the capabilities of any one method, and comparison of solutions allows students to develop their own system of approaches to problems, develops their intuition, and provides the necessary experience.

Let us present extremum problems admitting various solutions.

Example 1. Find the smallest value of an expression:

$$\sqrt{1+x^2-x}+\sqrt{1+x^2-x\sqrt{3}}$$

Solution. 1 way: geometric method.

It is clear that the smallest value of this expression will be at $x > 0$. Take a right angle with apex A and set aside the segments $AB = AC = 1$ on its sides (Fig.1.)

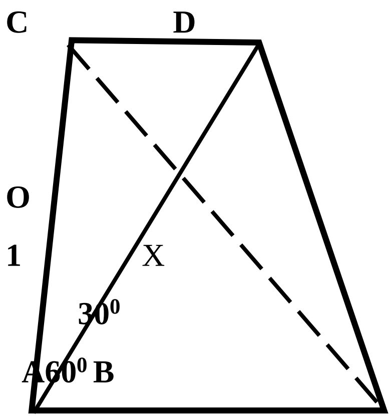

Fig.1

Having drawn a ray through point A inside the corner, forming angles of 60° and 30° with its sides, we put on this ray the segment $AD = x$. By the cosine theorem, we find that the first term is equal to BD , and the secondis equal to CD , i.e. this expression is equal to $BD + CD$. It will be the smallest when D lies on the segment BC, i.e. the smallest value is $\sqrt{2}$. For what values of x does this expression have the smallestvalue $\sqrt{2}$. The area of a right-angled triangle ABC is equal to the sum of the areas of triangles AOB and AOC:

$$\frac{1}{2}=\frac{1}{2}\cdot 1\cdot xSin60^\circ+\frac{1}{2}\cdot 1\cdot xSin30^\circ$$

$$1 = x\cdot\frac{\sqrt{3}}{2}+\frac{x}{2}, x=\frac{2}{\sqrt{3}+1}=\sqrt{3}-1.$$

2 way: coordinate method.

$$\sqrt{1+x^2-x}+\sqrt{1+x^2-x\sqrt{3}}=\sqrt{\left(x-\frac{1}{2}\right)^2+\frac{3}{4}}+\sqrt{\left(x-\frac{\sqrt{3}}{2}\right)^2+\frac{1}{4}}$$

$$A=\left(\frac{1}{2};\frac{\sqrt{3}}{2}\right), B=\left(\frac{\sqrt{3}}{2};\frac{1}{2}\right), B'=\left(\frac{\sqrt{3}}{2};-\frac{1}{2}\right), C=(x;0)$$

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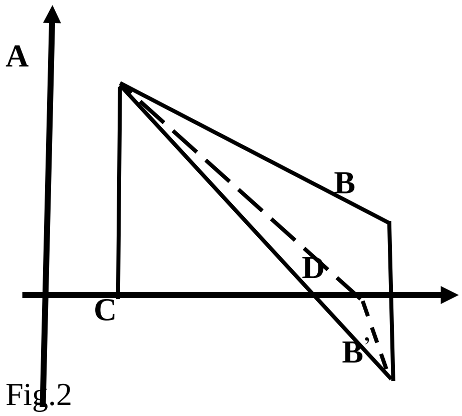


Fig.2

$AD + DB' \geq AB'$, the smallest value equal to the segment AB'.

$$AB' = \sqrt{\left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right)^2} = \sqrt{1+1}$$
$$= \sqrt{2}$$

From the collinearity of vectors $\overline{AC} = \overline{CB'}$

$$\frac{x - \frac{1}{2}}{\frac{\sqrt{3}}{2} - x} = \frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}}, \qquad \frac{2x-1}{\sqrt{3} - 2x} = \sqrt{3}, \qquad 2x - 1 = 3 - x \cdot 2\sqrt{3},$$

$$2x(\sqrt{3}+1) = 4, \qquad x = \frac{2}{\sqrt{3}+1} = \sqrt{3} - 1$$

3 way: <u>method of mathematical analysis.</u>

Now we apply the differential calculus, for this we represent the given expression as a functionof

$$f(x) = \sqrt{1 + x^2 - x} + \sqrt{1 + x^2 - x\sqrt{3}}$$

$$f'(x) = \frac{2x-1}{2\sqrt{1+x^2-x}} + \frac{2x - \sqrt{3}}{2\sqrt{1+x^2-x\sqrt{3}}} = 0,$$

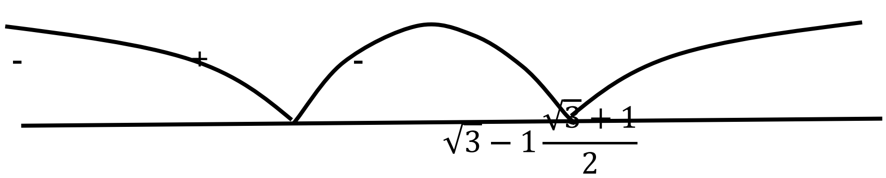
$$(2x-1)^2 \cdot (1 + x^2 - x\sqrt{3}) = \left(2x - \sqrt{3}\right)^2 \cdot (1 + x^2 - x)$$

$$4x^2 + 4x^4 - 4\sqrt{3}x^3 - 4x - 4x^3 - 4\sqrt{3}x^2 + 1 + x^2 - x\sqrt{3} =$$

$$= 4x^2 + 4x^4 - 4x^3 - 4\sqrt{3}x - 4\sqrt{3}x^3 - 4\sqrt{3}x^2 + 3 + 3x^2 - 3x$$

$$-2x^2 + (3\sqrt{3} - 1)x - 2 = 0$$

$$x_1 = \sqrt{3} - 1, x_2 = \frac{\sqrt{3}+1}{2}$$



$$x_{min} = \sqrt{3} - 1, f_{min}(x) = f(\sqrt{3} - 1) = \sqrt{2}$$

Comparing the above three solutions, we notice that the first is distinguished by the naturalness of reasoning, while the second is somewhat artificial, and the third is a classical method.

Example 2. The real numbers $x_1, x_2, \ldots, x_n$ belong to the segment [-1;1], and the sum of the cubes of these numbers is 0. Find the largest value of the sum:

$$x_1, +x_2+\ldots+x_n$$

1 way: algebraic method. Consider a polynomial satisfying the conditions of the problem

$$P(x) = 4(x+1)\left(x-\frac{1}{2}\right)^2 = (x+1)(4x^2-4x+1) = 4x^3-3x+1 \geq 0$$

$$P(x_1) = 4x_1^3 - 3x_1 + 1 \geq 0$$

$$P(x_2) = 4x_2^3 - 3x_2 + 1 \geq 0$$

…………………………..

$$P(x_n) = 4x_n^3 - 3x_n + 1 \geq 0$$

Adding the n obvious inequalities $P(x_i) \geq 0$, where $i = 1,2,\ldots,n$, we obtain

$$4(x_1^3+\ldots+x_n^3) - 3(x_1 + x_2+\ldots+x_n) + n \geq 0$$

by the condition of the problem

$$x_1^3 + x_2^3+\ldots+x_n^3 = 0$$

then we get the following inequality

$$3(x_1, +x_2+\ldots+x_n) + n \geq 0$$

where

$$x_1, +x_2+\ldots+x_n \leq \frac{n}{3}$$

2 way: trigonometric method.

We put that $x_i = Cos\varphi_i, i = 1,2,\ldots,n$.

We know that $Cos3\varphi = 4Cos^3\varphi - 3Cos\varphi$ from this formula we get that

$$4Cos^3\varphi_i - 3Cos\varphi_i\ , i = 1,2,\ldots,n.$$

$$4Cos^3\varphi_1 - 3Cos\varphi_1 \geq -1$$

$$4Cos^3\varphi_2 - 3Cos\varphi_2 \geq -1$$

………………………

$$4Cos^3\varphi_n - 3Cos\varphi_n \geq -1$$

Where

$$4(Cos^3\varphi_1+\ldots+Cos^3\varphi_n) - 3(Cos\varphi_1+\ldots+Cos\varphi_n) \geq -n$$

$$4(x_1^3+\ldots+x_n^3) - 3(x_1+\ldots+x_n) \geq -n$$

$-3(x_1 + \ldots + x_n) \geq -n$, or

$$x_1 + \ldots + x_n \leq \frac{n}{3}$$

Comparing the two solutions presented, we notice that the second is distinguished by the naturalness of reasoning, while the first is somewhat artificial. However, the first solutionis clearer.

Example 3. Find the extrema of the function $f(x, y) = 3x + 4y$ in condition

$$x^2 + y^2 = 16.$$

1 way: <u>algebraic method</u>. Let us calculate the sum of the following two terms

$$(3x + 4y)^2 + (3x - 4y)^2 = 9x^2 + 24xy + 16y^2 + 9x^2 - 24xy + 16x^2 = 25x^2 + +25y^2$$
$$= 25(x^2 + y^2) = 25 \cdot 16 = 400$$

$$(3x + 4y)^2 = 400 - (3y - 4x)^2, |3x + 4y| \leq 20$$

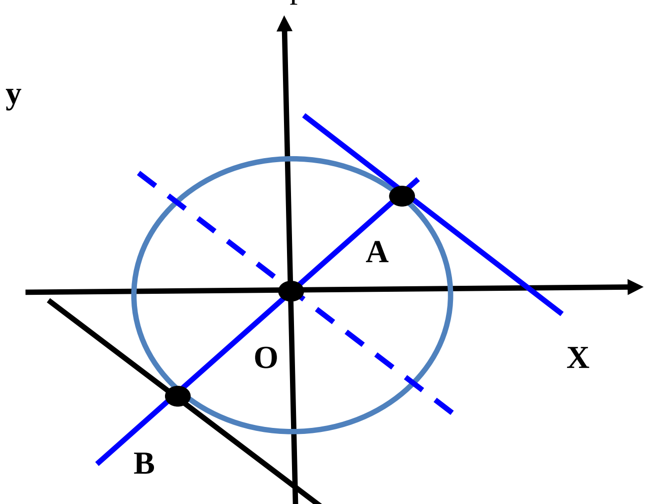
from here

$$-20 \leq 3x + 4y \leq 20, \begin{cases} y = \frac{4}{3}x & x = \pm 2{,}4 \\ x^2 + y^2 = 16 & y = \pm 3{,}2 \end{cases}$$

means

$$f_{max} = f(2{,}4; 3{,}2) = 20$$
$$f_{min} = f(-2{,}4; -3{,}2) = -20$$

2 way: <u>geometric method.</u> The level lines of the function $f(x, y) = 3x + 4y$ are parallel straight lines with the slope k$=\frac{3}{4}$. Obviously, the minimum is attained aZ point B, and the maximum aZ point A of the tangency of the level line and the circle $x^2 + y^2 = 16$. Letus find the coordinates of the point A and B.



ℓFig.3

To do this, it is enough to make the equation of the straight line "ℓ" and solve the system, consisting of the erosion of the straight line the equation of the circle. Note that line "ℓ" perpendicular to the level line, and. Therefore, its slope K_1 is $\frac{4}{3}$ ($K_1 K = -1$).

Line ℓ passes through point O and has a coefficient $K_1 = \frac{4}{3}$. Therefore, its equation is as follows: $y = \frac{4}{3}x$. Solving the system

$$\begin{cases} x^2 + y^2 = 16 \\ y = \frac{4}{3}x, \end{cases}$$

we get

$x = \pm 2{,}4,\ \ y = \pm 3{,}2.$

So, a minimum of -20 is reached at point $B = (-2{,}4; -3{,}2)$, and amaximum of 20 is reached at point $A = (2{,}4; 3{,}2)$.

3 way: <u>trigonometric method</u>. We know that the equation of the circle in parametric form has $x = RCost, y = RSint$, where R is the radius of the circle. Then our function has the from

$$3x + 4y = 3 \cdot 4Cost + 4 \cdot 4Sint = 12Cost + 16Sint = \sqrt{12^2 + 16^2}Sin(t + \varphi)$$

where

$$\varphi = arctg\frac{12}{16} = arctg\frac{3}{4}$$

Means, $3x + 4y = 20Sin(t + \varphi)\ , -1 \le Sin(t + \varphi) \le 1, -20 \le 3x + 4y \le 20.$

Problem solved.

4 way: <u>The method of Lagrange Multipliers.</u>

Suppose $f(x, y)$ and $g(x, y)$ are functions whose first-order partial derivatives exist. To find the relative maximum and relative minimum of $f(x, y)$ subject to the constraint that $g(x, y) = k$ for some constant k, introduce a new variable λ (the Greek letter lambda) and solve the following three equations simultaneously:

$$f_x^{'}(x, y) = \lambda g_x^{'}(x, y), f_y^{'}(x, y) = \lambda g_y^{'}(x, y), \mathrm{g(x, y) = k}$$

The desired relative extrema will be found among the resulting points (x, y).

Now let’s return to our task

$$f(x, y) = 3x + 4y, g(x, y) = x^2 + y^2, x^2 + y^2 = 16.$$

Use the partial derivatives

$$f_x^{'} = 3, f_y^{'} = 4, g_x^{'} = 2x, g_y^{'} = 2y$$

to write the three Lagrange equations you get

$\lambda = \frac{3}{2x}$ and $\lambda = \frac{4}{2y}$

(since $y \neq 0\ and\ x \neq 0$), which implies that

$\frac{3}{x} = \frac{4}{y}$ or $y = \frac{4}{3}x$

ISSN: 2249-877X Vol. 11, Issue 8, August 2021, Impact Factor: SJIF 2021= 7.642

$$\begin{cases} x^2+y^2=16 \\ y=\frac{4}{3}x, \end{cases}$$

from here

$x=\pm 2{,}4,\ \ y=\pm 3{,}2.$

$$f_{min}=f(-2{,}4;-3{,}2)=-20,\quad f_{max}=f(2{,}4;3{,}2)=20.$$

In the third example, the first, second and third methods are particular methods of solving problems. The fourth way is a general technique for solving problems, based on methods of mathematical analysis.

Example 4. If the product of two positive numbers is not less than their sum, then find the smallest value of their sum.

Solution.1 way: <u>Inequality method</u>. This problem has many different solutions using simple inequalities for two positive numbers:

$\frac{x+y}{2}\geq\sqrt{xy},\ \frac{x}{y}+\frac{y}{x}\geq 2$ and etc.

We write the condition $ab\geq a+b$ as follows:

$$(a-1)(b-1)\geq 1$$

Both parentheses must be positive (after all, $0<a<1$, then $(a-1)(b-1)<1$). Then, according to the inequality between the arithmetic and geometric means $a-1$ *and* $b-1$, we have

$$a-1+b-1\geq 2\sqrt{(a-1)(b-1)}\geq 2.$$

From here $a+b\geq 4$. Hence, the smallest value of numbers is 4.

2 way: <u>Inequality method</u>. This condition is equivalent to the fact that theharmonic mean of the numbers a *and* b is not less than 2:

$$\left(\frac{a^{-1}+b^{-1}}{2}\right)^{-1}\geq\ \frac{2ab}{a+b}\geq 2$$

But the arithmetic mean is less than the harmonic mean:

$\frac{a+b}{2}\geq\ \frac{2ab}{a+b}\geq 2.$

From here

$$a+b\geq 4.$$

3 way: <u>Inequality method</u>. Dividing this condition by a *and* b, we get

$$a\geq\frac{a}{b}+1, b\geq\frac{b}{a}+1$$

whence

$$a + b \geq \frac{a}{b} + \frac{b}{a} + 2 \geq 4$$

4 way: <u>Inequality method</u>.

Put $S = a + b$

$$\frac{(a+b)^2}{4} \geq ab \geq a + b, for\ \ S = a + b$$

we get

$\frac{S^2}{4} \geq S$ or $S \geq 4 \Longrightarrow a + b \geq 4$

We suggest the reader to choose the cutest one to his taste.

CONCLUSION

One of the ways to establish connections between algebra and geometry is to use the geometric method in solving algebraic problems, which involves the construction of a geometric model of the problem and its analytical solution, which is based on exact geometric relations.

Solving extreme problems in different ways and choosing the most rational of them contributes to the development of creative thinking of students. The conducted pedagogical experiment confirmed the effectiveness of the developed methodology for teaching mathematics based on solving extreme problems by different methods leads to the intensification of the cognitive activity of students, the development of their creative abilities, and the improvement of integral ideas about mathematics and hermethods.

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