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FORCED VIBRATIONS OF A VISCOELASTIC THREE-LAYER PLATE

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ABSTRACT

In this article, forced vibrations of a viscoelastic three-layer plate of a particular type are considered and some solution of numerical computation by Maple is given. On the basis of the obtained refined equations of vibrations, the problem of harmonic vibrations of a trisyllabic plate is solved. Various methods and approaches are used to reduce the three-dimensional in spatial coordinates of the problem of the theory of plates to the two-dimensional one. The article is devoted to the development of the theory of symmetric vibrations of a three-layer elastic plate in a plane setting with respect to two unknown functions, which are the main parts of the displacements of some "intermediate" surface of the plate.

KEYWORDS: *Three-Syllable Plate, Forced Vibrations, Viscoelastic, Frequency Equations, Stress-Strain State, Displacement.*

INTRODUCTION

The main requirements of scientific and technological progress in the field of construction, especially in earthquake-prone regions of the country, are aimed at increasing the durability and reliability of civil and industrial buildings and structures, the use of modern calculation methods using information technology. Consequently, these requirements lead to the need to improve the state of construction science and must meet the increased requirements of construction practice.

Composite materials in the form of a plate have found wide application in various fields of technology and construction.This is due to the fact that the lightness and rationality of the shape inherent in thin-walled structures is combined with their high bearing capacity, efficiency and good manufacturability.The theory of plates is one of the most relevant sections of the applied theory of elasticity and viscoelasticity.From this set of questions we will consider only the simplest and most important tasks in practice.Proceeding from the exact three-dimensional formulation of the problem and its solution in transformations, general equations of vibrations of three-syllable plates are derived, from which one can obtain the type of classical equations of vibrations. On the basis of the obtained refined equations of vibrations, the problem of harmonic vibrations of a trisyllabic plate is solved.Various methods and approaches are used to reduce the three-dimensional in spatial coordinates of the problem of the theory of plates to the two-dimensional one.

LITERATURE REVIEW

In this case, the displacements of the middle surface of the plate **[1]** are taken as the main unknown functions, and various simplifying hypotheses and prerequisites of a mechanical and geometric nature are applied **[2].** The hypotheses and assumptions used in the construction of the theory, together with simplifications, lead to significant disadvantages and errors.In the theory of Kirchhoff-Laméshells, these disadvantages are significant.

At one time, V.V. Novozhilov and R.M. Finkelstein **[2],** H.M. Mushtari **[3],** V.M. Darevsky **[4],** U.K. Nigul **[5],** therefore, "a more careful observance of the Kirchhoff – Lamé hypotheses still does not guarantee obtaining more accurate oscillation equations" **[6].**

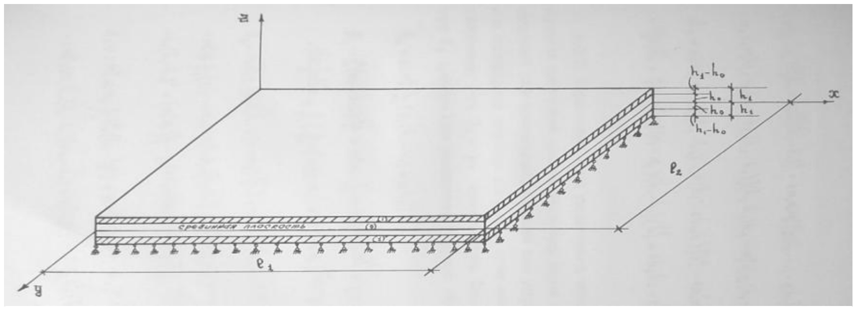
The dynamic calculation of multilayer, in particular, three-layer, plates in many cases is based on classical theories, which are based on Kirchhoff's hypotheses or refined theories of the Timoshenko type **[7].** Over the past several decades, the theory of plate vibrations has been developed based on the method of exact solutions by G.M.Petrashenya **[8].** This method was used to develop various versions of the theory of oscillation of three-layer plates of symmetric structure I.G.Filippov and his students **[9, 10].** In them, when deriving the equations of oscillation, the main parts of the components of the displacements of the points of the median surface of the filler are taken as unknowns, the number of which in the general case is six.If the boundary conditions are formulated exactly, then the number of unknowns will increase, according to the authors themselves, to twelve **[11,12].** The article is devoted to the development of the theory of symmetric vibrations of a three-layer elastic plate in a plane setting with respect to two unknown functions, which are the main parts of the displacements of some "intermediate" surface of the plate. An algorithm for determining the SSS of a plate in its arbitrary section has been developed. **[13,14]**

DISCUSSION

Consider in the Cartesian coordinate system an infinite isotropic three-layer plate $-\infty \le (x, y) \le \infty$; $-h_1 \le z \le h_1$. Suppose when the upper layers are viscoelastic and the inner layer is elastic. It is also accepted that the contacts between the bearing layers and the core are rigid. Taking into account the unlimited size of the plate, in what follows, we assume that it is in plane deformation, i.e. we refer it to the system of rectangular coordinates Oxz (Fig. 1).

In this case, the Ox axis is directed along the cross section Oxz along its midline, and the Oz axis - upward. Let us number the layers of the plate as in Fig. 1, i.e. the top bearing layer will be called the first layer, the bottom bearing layer the second and the filler layer zero. Let h_1, $2h_0$ and h_2 be the thicknesses of the first, zero and second layers. In this case, the viscoelastic operators for the inner layers are replaced by the elastic Lamécoefficients λ_1, μ_1 respectively.

Fig. 1.Research object



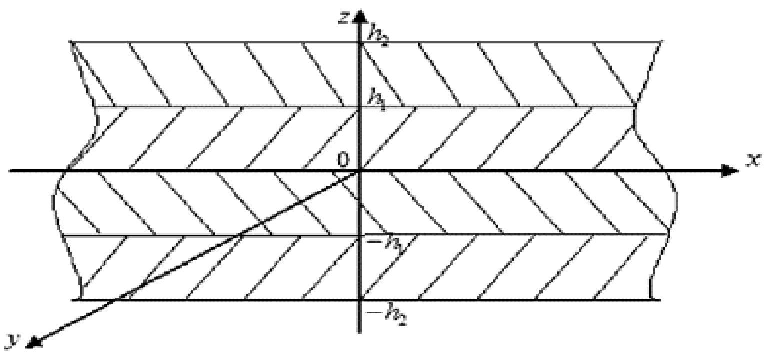


Fig. 1.Research object

Such a three-layer plate will be considered a layered medium, with the parameters of the material of the middle layer denoted by the index "O", and the parameters of the upper and lower layers by the index "I".

Dependences of stresses on deformations are taken in the form:

$$\begin{cases} \sigma_{ij}^{(l)} = L_e\left(\varepsilon^{(l)}\right) + 2M_e\left(\varepsilon_{ij}^{(l)}\right); \\ \sigma_{ij}^{(l)} = M_e\left(\varepsilon_{ij}^{(l)}\right); \quad \left(i \neq j \; ; i, j = x, y, z\right) \end{cases} \tag{1}$$

where L_l and M_l are viscoelastic operators:

$$\begin{cases} L_e\left(e_l\right) = \lambda_l\left[e_l\left(t\right) - \int_0^t f_{il}\left(t-\xi\right)\varsigma\left(\xi\right)d\xi\right]; \\ M_e\left(e_l\right) = \mu_l\left[\varsigma\left(t\right) - \int_0^t f_{2l}\left(t-\xi\right)\varsigma\left(\xi\right)d\xi\right]; \end{cases} \tag{2}$$

f_{kl} are kernels of viscous operators, λ_l , μ_l are elastic constants or Lamé coefficients.

By introducing the potentials $\Phi^{(l)}$ and $\overrightarrow{\psi}^{(l)}$ according to formula

$$\overrightarrow{U}^{(l)} = grad\Phi^{(l)} + rot\overrightarrow{\psi}^{(l)}; \; \overrightarrow{U}^{(l)} = U^{(l)}\left(u^{(l)}, v^{(l)}, w^{(l)}\right)$$

with the condition $div\vec{\psi} = 0$, the equations of motion of the material of the layers are reduced to the form

$$N_1\left(\Delta\Phi^{(l)}\right) = \rho_l \frac{\partial^2\Phi^{(l)}}{\partial t^2} \; ; \; M_l\left(\Delta\vec{\psi}^{(l)}\right) = \rho_l \frac{\partial^2\psi}{\partial t^2} \; ; \; N_l = L_l + 2M_l \qquad (4).$$

The boundary conditions on the surface of a three-layer plate are as follows:

At $z = \pm h_1$ (on the plate surfaces)

$$\sigma_{zz}^{(1)} = f_z^{\pm}(x,y,t) \; ; \; \sigma_{xz}^{(1)} = f_{xz}^{\pm}(x,y,t) \; ; \; \sigma_{yz}^{(1)} = f_{yz}^{\pm}(x,y,t) \; ; \qquad (5)$$

At $z = \pm h_0$ (contact plane):

$$\begin{cases} \sigma_{zz}^{(1)} = \sigma_{11}^{(0)} \; ; \sigma_{xz}^{(1)} = \sigma_{xz}^{(0)} \; ; \sigma_{yz}^{(1)} = \sigma_{yz}^{(0)} \; ; \\ U^{(1)} = U^{(0)}; V^{(1)} = V^{(0)}; W^{(1)} = W^{(0)}; \end{cases} \qquad (6)$$

The initial conditions are zero.

Solution of equations in the form

$$W_{n,m}(t) = W_0 \exp\left(\frac{b_1}{h_1}\xi t\right) \qquad (7)$$

Where ξ is the dimensionless complex frequency of the three-layer plate.

Substituting expression (7) into the equation of complex frequency ξ we obtain an algebraic equation of the fifth order:

$$f(\xi_j) = \xi^5 + B_1\xi^4 + B_2\xi^3 + B_3\xi^2 + B_4\xi + B_5 = 0 \qquad (8)$$

To solve equation (8), we introduce dimensionless parameters:

as in the previous paragraph, the coefficients B_j take the form:

$$B_1 = \frac{C\{b^2\rho[\rho(4-D_0)\frac{h}{2} + (1-h)](1-h)h + 2b^4\rho^2[1+(1-D_1)(1+2h)/3](1-h)^2/2\}}{\{b^4\rho^2[1+(1-D_1)(1+2h)/3](1-h)^2/2 + b^2\rho[\rho(4-D_0)\frac{h}{2} + (1-h)](1-h)h + \rho^2(4-D_0)\frac{h^3}{6}\}} \; ;$$

$$B_2 = \frac{\{b^4\rho^2[(1-h)+\rho h] + C^2b^2\rho[1+(1-D_1)\frac{(1+2h)}{3}]\frac{(1-h)^2}{2} + b^2\rho[\rho[D_0 + (2+D_0)\frac{h}{3}]h^2 - 4D_1[(1-h)+\rho h]}{\{b^4\rho^2[1+(1-D_1)(1+2h)/3](1-h)^2/2 +} \; ;$$

$$\frac{\times(1-h)h]\gamma+b^4\rho^2[2(1-h)^2-\rho h^2](1-h)-(1-D_1)[(1-h)^2[(1+h)+\frac{(1-h)}{3}]-\rho(1-h)(2-h)h]]\gamma\}}{+b^2\rho[\rho(4-D_0)h/2+(1-h)](1-h)h+\rho^2(4-D_0)h^3/6\}};$$

$$B_3=\frac{C\{b^4\rho^2[(1-h)+\rho h]+b^4\rho^2[[2(1-h)^2-\rho h^2](1-h)-(1-D_1)[2(1-h)^2(1+2h)/3-\rho(1-h)(2-h)h]]\gamma\}}{\{b^4\rho^2[1+(1-D_1)(1+2h)/3](1-h)^2/2+b^2\rho[\rho(4-D_0)\frac{h}{2}+(1-h)](1-h)h+\rho^2(4-D_0)\frac{h^3}{6}\}};$$

$$B_4=\frac{\{4D_1b^4\rho^2[(1-h)^2-3h](1-h)/3+b^6\rho^3[4D_0h/3-(1-h)]h^2\}\gamma^2}{\{b^4\rho^2[1+(1-D_1)\frac{(1+2h)}{3}]\frac{(1-h)^2}{2}+b^2\rho[\rho(4-D_0)\frac{h}{2}+(1-h)](1-h)h+\rho^2(4-D_0)\frac{h^3}{6}\}};$$

$$B_5=\frac{C\{4D_1b^4\rho^2[(1-h)^2-3h](1-h)/3+b^6\rho^3[4D_0h/3-(1-h)]h^2\}\gamma^2}{\{b^4\rho^2[1+(1-D_1)\frac{(1+2h)}{3}]\frac{(1-h)^2}{2}+b^2\rho[\rho(4-D_0)\frac{h}{2}+(1-h)](1-h)h+\rho^2(4-D_0)\frac{h^3}{6}\}}; \qquad (9)$$

Equation (8) was solved numerically using the Maple 7 software.

In this case, the calculations were carried out for various materials of the plate with the following values of their physicomechanical parameters:

- steel: $E=2\cdot10^{11}\,Pa$; $\rho=7850\frac{kg}{m^3}$; $\nu=0.25$;
- copper: $E=10^{11}\,Pa$; $\rho=8940\frac{kg}{m^3}$; $\nu=0.31$;
- aluminum: $E=7\cdot10^{10}\,Pa$; $\rho=2750\frac{kg}{m^3}$; $\nu=0.35$.

For example: Let a three-layer plate have a shape wiew. The upper part 1 is made of steel material with Young's modulus $E=2\cdot10^{11}\,Pa$; Poisson's ratio $\nu=0.29$; and density $\rho=7850\frac{kg}{m^3}$. The lower part 2 is made of copper with. Young's modulus $E=10^{11}\,Pa$; Poisson's ratio $\nu=0.375$; and density $\rho=8940\frac{kg}{m^3}$.

RESULTS

The results of the calculations are shown in Fig. 2-3 in the form of the dependences of the lowest frequency $f(\xi)$ on the wave number ξ. Figure 2 shows the graphs of the dependences of the frequency on the wave number for different values of the plate thickness: $h=0.1;0.5$.

The graphs given (Figs. 2-3) show that for the values of the wave number $\xi<3$ the dependences of the frequency $f(\xi)$ on the wave number ξ are nonlinear, and later, with an increase in ξ, these dependences become linear. The values of the frequency of a thin plate at $h = 0.1$ differ sharply from the others, where the coefficients B_j are constant, depending on the geometric parameters and mechanical properties of the layers of the plate. Frequency equations were solved using Maple-7.11 programs with the following problem data for the bearing layers:

steel $\rho = 7850 \frac{kg}{m^3}$; $E = 2 \cdot 10^{11}\, Pa$, $\nu_1 = 0,29$; $0,375$; $\rho = 2750 \frac{kg}{m^3}$, $E = 10^{11}\, Pa$.

Wherein $h_1 = 0.1m$, $h_2 = 0.5m$, $\gamma = 5$.

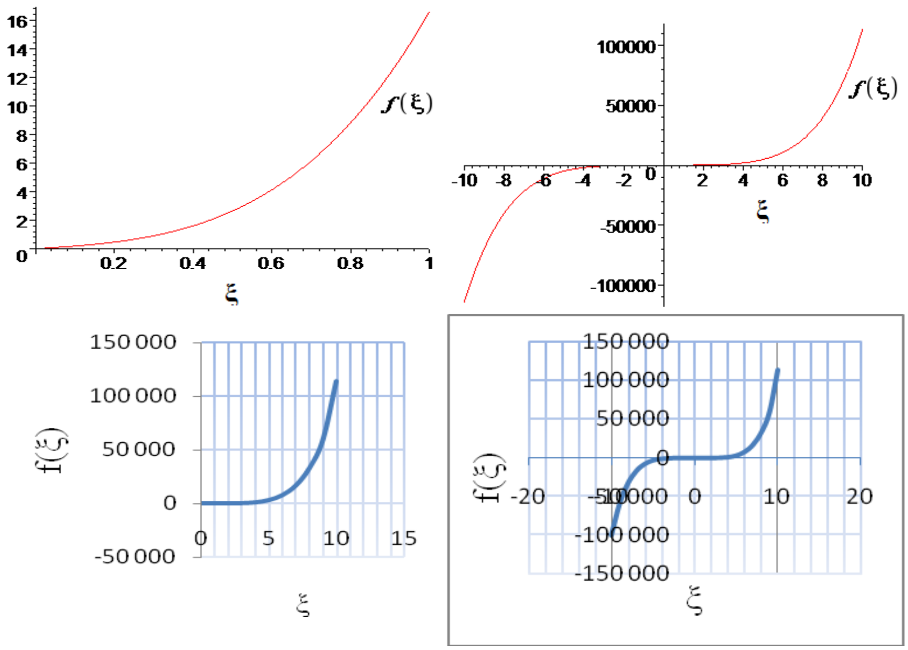


Fig. 2. Curves of change $f(\xi)$ depending on
$D_0 = 0,15; D_1 = 0,15;\ h = 0,1;\ b = 0,5;\ \rho = 0,5;\ C = 0,001;\ \gamma = 5.$

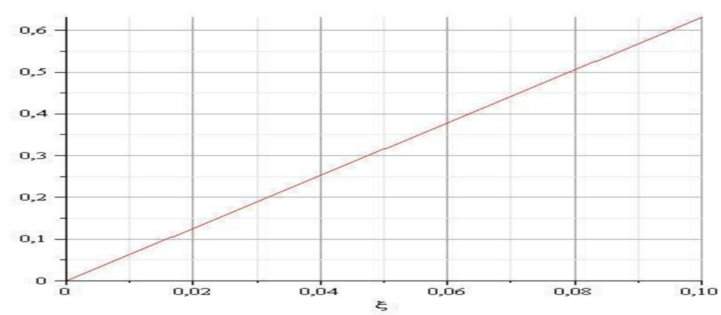


Fig. 3. Curves of change $f(\xi)$ depending on ξ.
$D_0 = 0,5; D_1 = 0,5;\ h = 0,1;\ b = 0,5;\ \rho = 0,5;\ C = 0,001;\ \gamma = 5.$

Algebraic equation (8) is solved numerically on Maple 7,11 and Excel in exactly the same way as in the case of the first problem.

As a result of calculating the attenuation coefficients for the values of the parameters $h = 0{,}1; 0{,}5;\ b = 0{,}5;\ \rho = 0{,}5;\ \nu_0 = 0{,}29;\ 0{,}375;\ \nu_1 = 0{,}29;\ 0{,}375;\ C = 0{,}001$

$$0{,}1 \le \gamma \le 5.$$

Here ν_0, ν_1 – is the Poisson's ratio of the material of the layers of the plate.

CONCLUSION

Thus:

- Consideration of a three-layer plate as a three-dimensional body in an exact three-dimensional formulation makes it possible to derive general and approximate equations of vibration of three-layer plates of a particular type without invoking any hypotheses;

- From the general equations, it is possible to derive approximate equations of oscillation of any finite order in derivatives suitable for solving particular applied problems;

- In the limiting cases, the obtained approximate equations transform into the well-known classical equations for plates describing longitudinal or transverse vibrations;

- The described approach made it possible not only to obtain the equations of oscillation of a three-layer plate, but also formulas for calculating all displacements and stresses at the points of a three-layer plate through the sought functions;

- The obtained general and approximate equations explicitly contain viscoelastic operators describing the rheological behavior of the material of a three-layer plate;

- The particular applied problems presented in the work for a three-layer plate made it possible to evaluate the influence of various parameters on the stress-strain state of the plate. Frequency equations are obtained for longitudinal and transverse vibrations of a three-layer plate taking into account the dispersion of waves. The effect of viscosity on the wave field in the plate is shown.

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