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MATHEMATICAL MODEL FOR PREDICTION OF GROUNDWATER LEVELS IN TWO-LAYER FORMATIONS

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ABSTRACT

A article discusses the process of forecasting changes in the level of ground and pressure water. A brief analysis and computational experiments of scientific papers on mathematical and numerical modeling of the object under study are given. For a comprehensive study of the problem under consideration, a mathematical model was developed that takes into account the external source, evaporation, filtration coefficients, active porosity, filtration rate and two-way boundary conditions. An effective numerical algorithm has been developed for predicting changes in the ground water level using a combination of finite-difference schemes and run-through methods. It has been studied that changes in the level of ground and pressure water, filtration permeability, water loss coefficient and filtration rate associated with the water level can have a serious impact on the environmental process.

KEYWORDS: *Groundwater Abstraction, Salt Transfer, Mathematical Model Of Filtration, Desalination Technological Schemes, Geofiltration Process.*

INTRODUCTION

The main task in the issues of sustainable development of the agricultural sector is to increase crop yields and the quality of the output product while observing significant savings in labor and energy resources, and environmental protection requirements. This, in turn, is connected to solving the problems of substantiating the intensity of water reclamation of agricultural lands, optimizing calculations of agricultural drainage and managing the water regime of agricultural lands. It should be noted that the volume of drainage and field waste waters in many irrigation systems in Central Asia, Transcaucasia and other regions reaches $30\ \%$ of the water intake.

These problems can be investigated and elaborated with the effective mathematical tool - "model-algorithm-software product-computational experiment", which makes it possible to monitor and predict changes in the groundwater level with subsequent management decisions.

Such a mathematical tool can be effectively used for quantitative assessment of filtration conditions in a complex natural environment, for a detailed study of regional patterns of formation, distribution, and flow of groundwater, and for scientific substantiation of methods and volumes of planned hydro geological studies [1-3].

When developing a mathematical model for the interaction of ground and surface waters, the basic laws of hydrodynamics and systems of linear and nonlinear partial differential equations with appropriate boundary conditions are used [4-5].

The foundations of the science of groundwater flow (hydrogeology) are connected to the names of A. Darcy, J. Dupuy, N.E. Zhurkovsky, F. Forchheimer and others.

An important role in the development of mathematical methods with the intensive development of the theory and practice of groundwater flow was played by the studies of F.B. Abutaliev, E.B. Abutaliev, P.Ya. Polubarinova-Kochina, V.I. Aravina, S.N. Numerova, N.N. Verigina, G.N. Kamensky, A.I. Silin-Bekchurin, P.P. Klimentov, G.B. Pykhachev, V.A. Mironenko, I.K. Gavich and others.

In particular, the study in [6] considers the issues of numerical modeling of hydrogeological processes in solving the problem of desalination of highly mineralized waters of the Amudarya River deposits, characterized by complex geofiltration and geomigration conditions.

F.B. Abutaliev and E.B. Abutaliev developed a mathematical complex for calculating and predicting changes in the groundwater level depending on the hydrodynamic parameters of the object of study [12].

The studies conducted by A.A. Tskhai, K.B. Koshelev, N.Yu. Kim [13] proposed a mathematical description of the movement of ground and surface waters based on geofiltration models [14-16].

In [17], a mathematical model, numerical algorithm and software tool were presented for research and forecasting, and making managerial decisions on the process of joint movement of ground and surface waters in a river basin.

A.A. Kashevarov in [18] proposed hydrodynamic and hydraulic models of water runoff in wetlands.

The author of [19] proposed models of the steady-state motion of groundwater in formations adjoining low permeable formations.

In [20], a two-dimensional stable flow of groundwater in the vertical plane was considered.

Formulation of the problem. The hydrogeological and hydrodynamic analysis of the process of groundwater movement showed that in the mathematical modeling of the geofiltration process, at least a two-layer medium should be considered, which consists of two layers: the ground layer (with low throughput) and water (Fig. 1).

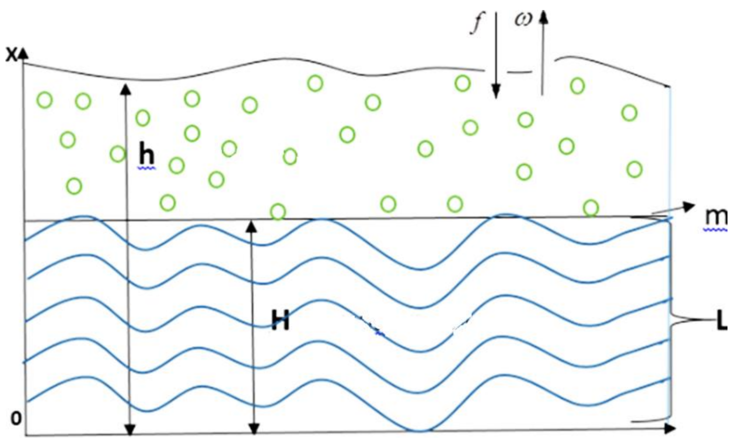

Fig.1. Geofiltration process

Any changes in the water level will lead to the interaction of the hydrodynamic and hydrochemical regimes of the two layers of groundwater (GW). Under these conditions, it is necessary to pay attention to the protection of the interaction forces of the layers from the penetration of mineralized waters through the boundaries of the water levels.

GW motion under these conditions is described by a system of partial differential equations:

$$\left.\begin{aligned} &\mu\frac{\partial h}{\partial t} = -k_b\frac{h-H}{m} + f - \omega; \\ &\mu^*\frac{\partial H}{\partial t} = -k\frac{H-h}{m} + \frac{\partial}{\partial x}\left(T\frac{\partial H}{\partial x}\right), \end{aligned}\right\} \qquad (1)$$

where $h(x,t)$, $H(x,t)$ are the levels of ground and pressure waters; μ μ^* are the coefficients of water loss; m is the thickness of the separating layer; k_b, k are the filtration coefficients of the upper and lower formations, T is the filtration conductivity of the main horizon, f is the external source, ω is evaporation.

System (1) is solved under the following initial and boundary conditions:

$$h(x,0) = h_0(x), \;\; H(x,0) = H_0(x), \qquad (2)$$

$$T\frac{\partial H}{\partial x}\bigg|_{x=0} = -\lambda(H - H_A),\; T\frac{\partial H}{\partial x}\bigg|_{x=L} = \lambda(H - H_B), \qquad (3)$$

where $h_0(x)$, $H_0(x)$ are the initial conditions for the levels of groundwater and pressure water, λ is the coefficient to reduce the boundary condition to the dimensional form, H_A, H_B are the boundary values of pressure waters.

Solution method. Since the problem is described by a system of quasi-linear differential equations, it is difficult to obtain its solution in an analytical form. For the numerical solution of problem (1)-(3), we use the method of finite differences, that is, we replace the differential operators in equations (1), (3) with finite-difference operators [7-11]. To do this, we introduce a grid into domain $D=\{0\le x<L_h,\ 0\le t\le T\}$, where T is the maximum time during which the process is being studied, we divide interval $[0,L_x]$ by step h, and $[0,T]$ by step τ; as a result, we obtain the following grid:

$$\omega_{h\tau}=\Big\{(x_i,t_j),\ i=0,1,2,...N\ \ j=0,1,...,J;$$
$$x_i=i\Delta x;\ \ t_j=j\tau;\ \tau=T/J,\ h=L_x/N\Big\}.$$

We approximate equation (1) using an implicit scheme on grid $\omega_{h\tau}$ in the following form

$$\left.\begin{aligned}&\frac{h_i^{j+1}-h_i^j}{\tau}=-\frac{k_b}{\mu\cdot m}(h_i^j-H_i^j)+\frac{f_i^j-\omega_i^j}{\mu};\\&\frac{H_i^{j+1}-H_i^j}{\tau}=-\frac{k}{\mu^*\cdot m}(H_i^{j+1}-h_i^{j+1})+\\&\qquad+\frac{T_{i+0.5}}{\mu^*\cdot\Delta x^2}(H_{i+1}^{j+1}-H_i^{j+1})-\frac{T_{i-0.5}}{\mu^*\cdot\Delta x^2}(H_i^{j+1}-H_{i-1}^{j+1});\end{aligned}\right\}\tag{4}$$

$$T_0\cdot\frac{H_1^{j+1}-H_0^{j+1}}{\Delta x}=-\lambda(H_0^{j+1}-H_A);\tag{5}$$

$$T_N\cdot\frac{H_N^{j+1}-H_{N-1}^{j+1}}{\Delta x}=\lambda(H_N^{j+1}-H_B)\tag{5a}$$

Difference scheme (4) is reduced to a system of linear algebraic equations

$$h_i^{j+1}=\xi\cdot h_i^j-\xi_1\cdot H_i^j+\xi_2,\tag{6}$$

$$a_i\cdot H_{i-1}^{j+1}-b_i\cdot H_i^{j+1}+c_i\cdot H_{i+1}^{j+1}=-d_i^j.\tag{7}$$

Here

$$\xi=1-\frac{\tau\cdot k_b}{\mu\cdot m},\ \xi_1=-\frac{\tau\cdot k_b}{\mu\cdot m},\ \xi_2=\frac{\tau\cdot(f_i^j-\omega_i^j)}{\mu},$$

$$a_i=\frac{\tau\cdot T_{i-0.5}}{\mu^*\cdot\Delta x^2},\ b_i=\frac{\tau\cdot T_{i+0.5}}{\mu^*\cdot\Delta x^2}+\frac{\tau\cdot T_{i-0.5}}{\mu^*\cdot\Delta x^2}+\frac{k\cdot\tau}{\mu^*\cdot m}+1,\ c_i=\frac{\tau\cdot T_{i+0.5}}{\mu^*\cdot\Delta x^2},$$

$$d_i^j = -\frac{k \cdot \tau}{\mu^* \cdot m} \cdot h_i^{j+1} - H_i^j.$$

The solution to the systems of equations (6) is determined as follows:

$$H_i^{j+1} = \alpha_{i+1} H_{i+1}^{j+1} + \beta_{i+1}, \tag{8}$$

where the sweep coefficients are determined using the recursive relation:

$$\alpha_{i+1} = \frac{a_i}{b_i - a_i \cdot \alpha_i}, \ \beta_{i+1} = \frac{d_i^j + a_i \cdot \beta_i}{b_i - a_i \cdot \alpha_i}, \ i = 1, 2, 3, \ldots, N-1. \tag{8a}$$

For $i = 0$ equation (8) takes the following form (9)

$$H_0^{j+1} = \alpha_1 H_1^{j+1} + \beta_1. \tag{9}$$

as a result of simplification of (5) and (5a), we obtain

$$H_0^{j+1} = \frac{T_0}{\Delta x \lambda + T_0} H_1^{j+1} + \frac{\Delta x \lambda H_A}{\Delta x \lambda + T_0}, \tag{10}$$

$$H_{N-1}^{j+1} = \frac{T_N - \Delta x \lambda}{T_N} H_N^{j+1} + \frac{\Delta x \lambda H_B}{T_N}, \tag{11}$$

Comparing the values of equation (8) for $i = 0$ with (10), we find the sweep coefficients α_1 and β_1.

$$\alpha_1 = \frac{T_0}{\Delta x \lambda + T_0}, \ \beta_1 = \frac{\Delta x \lambda H_A}{\Delta x \lambda + T_0}.$$

Comparing the values of equation (8) for $i = N-1$ with (10), we find the level of pressure water H_N^{j+1}.

$$H_N^{j+1} = \frac{\beta_N T_N}{T_N - \Delta x \lambda - \alpha_N T_N} - \frac{\Delta x \lambda H_B}{T_N - \Delta x \lambda - \alpha_N T_N}$$

Machine algorithm of the computational process. The machine algorithm for solving the problem is realized as follows:

1 step. Input of initial (base) data (input of constants).

2 step. Calculation of the boundary values of the sought-for variables from boundary conditions of the problem.

3 step. Calculation of the elements of the tridiagonal transition matrix obtained as a result of the approximation of differential operators by finite difference operators.

4 step. Calculation of sweep coefficients (7).

5 step Calculation of the values of the sought-for variables of the task posed.

6 step. Checking for the adequacy of the task posed.

7 step. Interpretation of the results of the computational experiments conducted on a computer in the form of tabular and graphical objects.

Discussion of results

Based on the developed mathematical model (1)-(3) and its numerical algorithm, a software tool was compiled for conducting computational experiments on a computer, the results of which are shown in Figs. 2-8.

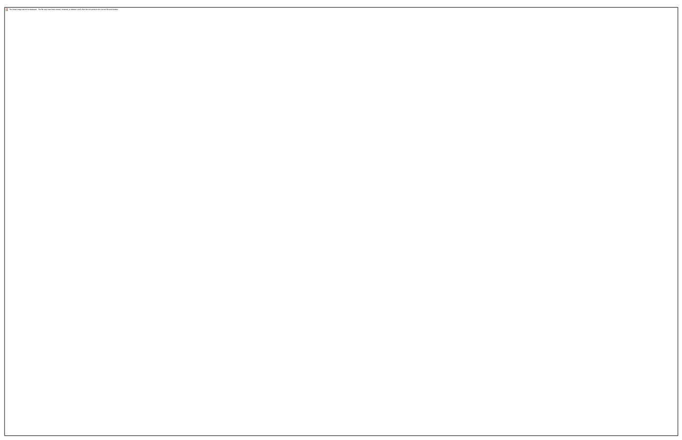

Fig. 2. Change in the level of groundwater and pressure water along the length of the layer (forecast for 30 days).

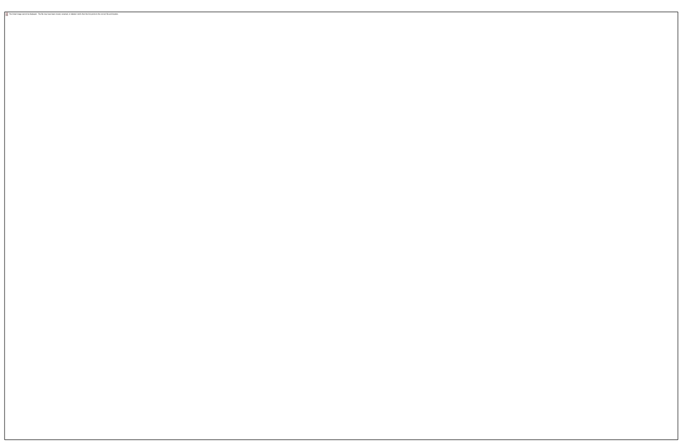

Fig. 3. Change in the level of groundwater and pressure water along the length of the layer (forecast for 90 days).

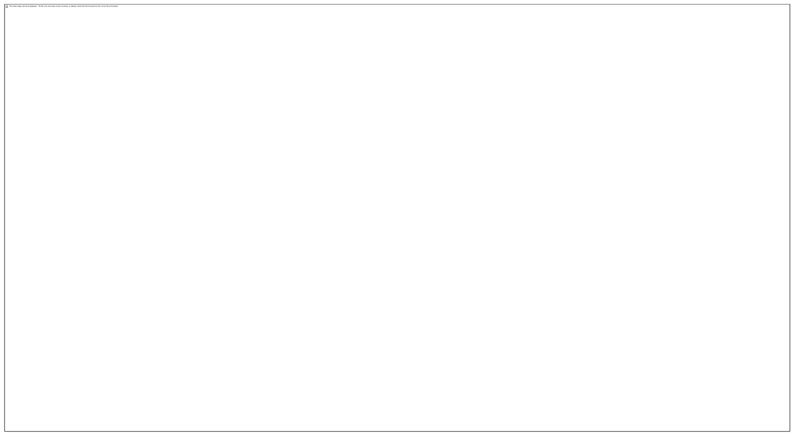

Fig. 4. Change in the level of groundwater and pressure water along the length of the layer (forecast for 150 days).

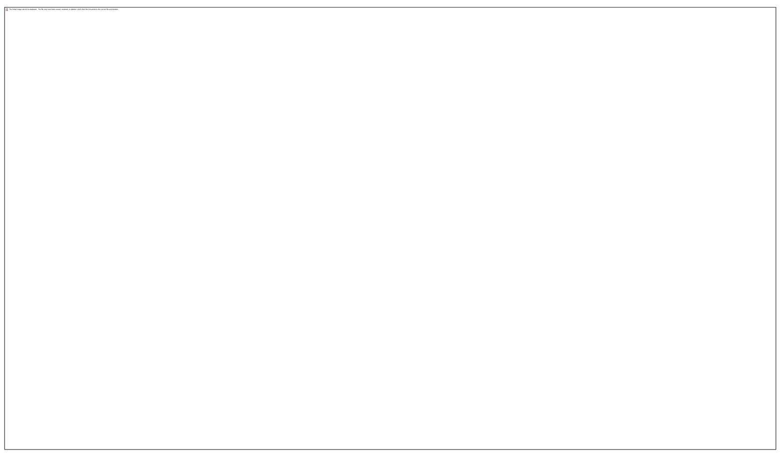

Fig. 5. Change in the level of groundwater and pressure water along the length of the layer (forecast for 180 days).

Figure 2 shows the results of changes in the levels of groundwater and pressure water over time for: $\mu^* = 0.3$, $\mu = 0.4$, $m = 4m$, $k_b = 0.9 m/day$, $k = 0.8\ m/day$, $T = 0.2 m^2/day$, $\Delta x = 0.05 км$, $t = 1\ month$, $f_i^j - \omega_i^j = 1\%$. As seen from the curves in Fig. 2, the groundwater level grows along the length according to a linear law, and the pressure water level grows slightly, but upon reaching the level $x \geq 0.4 km$, it increases sharply.

From the numerical calculations performed (Fig. 3), it follows that the groundwater level along the length of the layer remains almost unchanged, and the pressure water level is under-estimated for x changing from 0.01 to 0.36 *km*, but for $x \geq 0.42$ *km* it grows according to a parabolic law. This pattern continues for the forecast time *t=5 months* (Figs. 4-5).

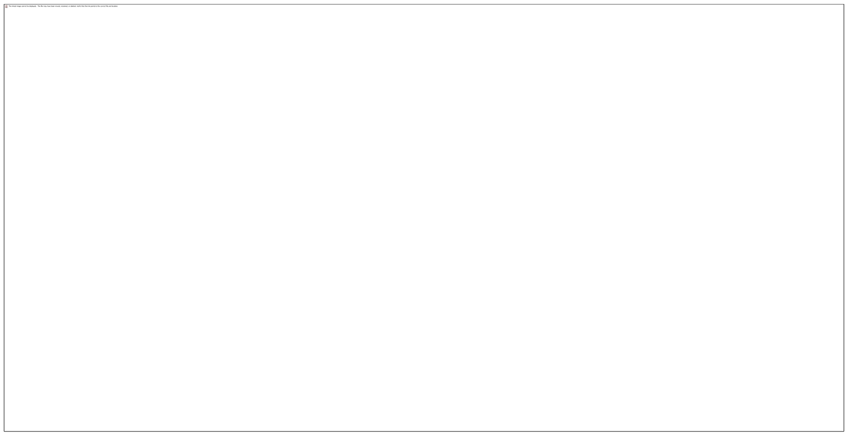

Fig. 6. Change in the groundwater level over time for $\mu^{*}=0.3,\quad \mu=0.4,\quad m=4m,$ $k_b=0.9m/day,\ \ k=0.8\ m/day,\ \ T=0.2m^2/day,\ \ \Delta x=0.5km,\ \ t=1;3;5;7\,month,$ $f_i^{\,j}-\omega_i^j=1\%.$

Fig. 7. Change in the groundwater level over time.

Numerical calculations performed on a computer (Fig. 6) show that the groundwater level decreases over time and becomes uniform in length. They show that this process significantly depends on the coefficients of filtration and water loss, the thickness of the separating layer, and external sources.

This pattern is observed in the change in the level of pressure waters along the length and over time (Fig. 7). As seen from the curves in Fig. 7, the change depends on the coefficient of elastic water loss, the thickness of the separating layer, the filtration coefficients, and the filtration conductivity of the main horizon. With an increase in the coefficients of filtration and water loss, the level of pressure water along the length of the layer begins to be uniform at all points.

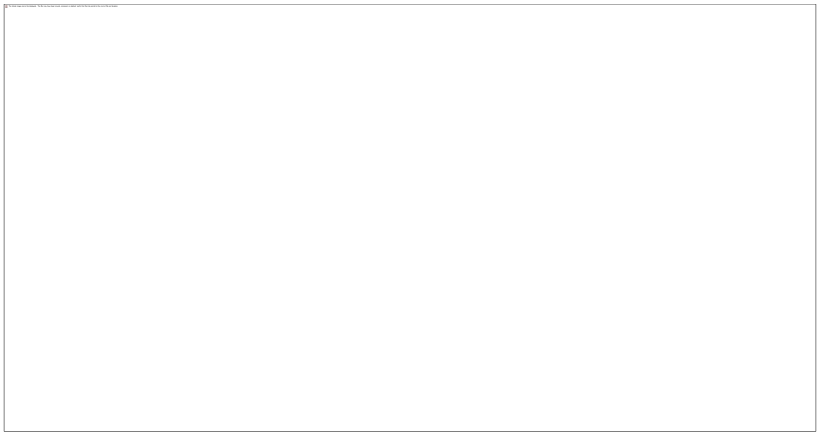

Fig. 8. Change in the level of groundwater for different values of filtration conductivity of the main horizon (at $T = 1; 5; 10 m^2 / day$).

Computational experiments (CE) conducted on a computer, for various values of filtration conductivity, showed that with an increase in the value of this parameter, the level of groundwater decreases along the length. This is especially seen at $T = 10 m^2 / day$ (Fig. 8).

CONCLUSIONS

An analysis of the CE conducted on a computer showed that the level of groundwater grows over time along the length according to a linear law, and the level of pressure waters changes insignificantly. Then, at $x \geq 0.4 km$ it grows sharply along the length of the considered area of groundwater filtration.

Numerical calculations have shown that the change in the groundwater level significantly depends on the coefficients of filtration and water loss, the thickness of the separating layer and external sources, and the level of pressure water depends on the coefficient of elastic water loss, the thickness of the separating layer, and the filtration coefficients and filtration conductivity of the main horizon.

CE conducted for different values of filtration conductivity showed that with an increase in the value of this parameter, the groundwater level decreases along the length.

REFERENCES

1. Klimintov P.P., Kononov V.M. Groundwater dynamics. - Moscow: Higher school, 1985. - 384 p.
2. Goldberg V.M. Hydrogeological forecasts of groundwater quality at water intakes. - Moscow.: Nedra. - 1976. - 153 p.
3. Akramov A.A. Artificial formation and replenishment of near-channel fresh water lenses. - Tashkent: Fan, 1989. - 196 p.
4. Konosavsky P.K., Soloveychik K.A. Mathematical modeling of geofiltration processes. - St. Petersburg: SPbGTU, - 2001. - 96 p.
5. Mironenko V.A. Groundwater dynamics. - Moscow: MGGU, 2001. - 519 p.

6. Usmanov R.N., Seitnazarov K.K. Software complex for fuzzy-deterministic modeling of hydrogeological objects // Automation and software engineering. - 2014. - No. 1 (7). – P. 9-22.

7. Kalitkin N.N., Koryakin P.V. Numerical methods: in 2 books. Book. 2. Methods of mathematical physics. - Moscow: Academy, 2013. - 304 p.

8. Remson J., Hornberger G.M. and Molz F.J. Numerical Methods in sub-surface Hydrology. – New York: WileyInterscience, 1971. – 389 p.

9. Miguela A., Luthin M., Luthin J. Seepage and Groundwater. – New York: Oxford Press, 1982. – 491p.

10. Holmes M. Introduction to Numerical Methods in Differential Equations. – New York: Troy, 2006. – 247 p.

11. Hornberger G. Numerical Methods in the Hydrological Sciences. – USA: American Geophysical Union, 2000. – 258 p.

12. Abutaliev F.B., Abutaliev E.B. Methods for solving problems of underground hydromechanics on a computer. - Tashkent, 1968. - 196 p.

13. Tskhai A.A., Koshelev K.B., Kim N.Yu. Model of interaction between underground and surface waters for a decision support system // Information systems in economics, ecology and education. - Barnaul: AltGTU, 2002. - P. 39-41.

14. Antontsev S.N., Epikhov G.P., Kashevarov A.A. System mathematical modeling of water exchange processes. - Novosibirsk: Nauka, 1986. - 215 p.

15. Zeegofer Yu.O., Klyukvin A.N., Pashkovsky I.S., Roshal A.A. Permanent models of the hydro-lithosphere of the territories of urban agglomerations (on the example of the Moscow agglomeration). - Moscow: Nauka, 1990. - 198 p.

16. Krashin I.I., Polshkov E.A., Orfanidi E.K. Automated grid models of groundwater basins. - Moscow, 1992. - 176 p.

17. Kosheleva E.D., Koshelev K.B. Numerical modeling of the joint movement of surface and underground waters in the decision support system // Bulletin of the Altai State Agrarian University. - 2005. - No. 3 (19). - P. 12-16.

18. Kashevarov A. A. Modeling of water runoff and salt transfer processes in wetlands, Applied Mechanics and Technical Physics. - 2005. - No. 1. - P. 96-105.

19. Polubarinova-Kochina, Ya. On the steady movements of groundwater in the formations adjoining low permeable formations // PMTM. - 1960. - No. 1. - P. 92-95.

20. Anderson E.I. An analytical solution representing groundwater–surface water interaction // Water Resource. Res. – 2003. – Vol. 39, Issue 3. – P. 1071. –DOI:10.1029/2002WR001536.