NUMERICAL MODELING TO CHANGE THE GROUND WATER LEVEL THE WATER AREA

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ABSTRACT

An urgent problem related to the process of changing the level of ground and pressure water is solved in the article; the problem is described by a system of partial differential equations and various corresponding initial, internal and boundary conditions. To derive a mathematical model of the process under consideration, a detailed review of scientific papers devoted to various aspects and mathematical support of the object of study is given. To conduct a comprehensive study of the process of filtering and changing the salt regime in groundwater, a mathematical model and an effective numerical algorithm are proposed taking into account external sources and evaporation. Since the process is described by a nonlinear system of partial differential equations, it is difficult to obtain an analytical solution. To solve it, a numerical algorithm based on a finite-difference scheme is developed, and an iterative scheme is used for nonlinear terms, which checks the convergence of iterative method.

KEYWORDS: *Mathematical Model, Numerical Algorithm, Groundwater, Ground And Pressure Water, Soil.*

INTRODUCTION

The main tasks of hydrogeology, including tasks related to land development, reclamation and irrigative construction, assessment of groundwater reserves and resources, and many others, ultimately provides for the prediction of the hydrodynamic and hydromechanical regime of groundwater - a closely interconnected element of a single geofiltration system. In particular, a decrease in the Aral Sea water level caused significant changes in the exploitation of aquifers, groundwater in the coastal zone, and had a negative impact on the environment. In fact, fisheries have been eliminated, the conditions of groundwater exploitation have worsened, the fauna has become poorer, and the surface freed from the sea is subjected to aeolian processes, which entail a decrease in the productivity of coastal pasture lands adjacent to the Aral Sea, etc.

To conduct a comprehensive study, forecast and managerial decisions on the above mentioned issue, a number of problems have been solved, where the core is a mathematical model, a numerical algorithm and software complex for conducting a computer experiment; significant theoretical and applied results have been obtained.

In [1], the stationary model of groundwater filtration is used to quantify and analyze underground hydrodynamics in the Akaki catchment, paying particular attention to the borehole field that supplies the city of Addis Ababa. The modeling is performed in a two-layer unlimited aquifer with a spatially variable recharge and hydraulic conductivity in well-defined boundary conditions. The model is used to predict the nature of the groundwater flow, the groundwater-surface water interaction, and the effect of pumping on borehole field in various scenarios.

In [2], a one-dimensional mathematical model of dissolved substances transport in finite aquifers is considered. The basic equation for the dissolved substances transport by an unsteady flow of groundwater is solved analytically by the Laplace transform method. Initially, the aquifer is subjected to a spatially dependent concentration of the source with zero order formation. One end of the aquifer receives the concentration of the source and is represented by a mixed-type boundary condition in a time domain of solution. The concentration gradient at the other end of the porous medium is assumed to be zero.

Mechanisms of artificial recharge affecting the groundwater reservoir were considered in [3]. Based on a generalized groundwater reservoir, various scenarios for the location of the infiltration basin and replenishment intensity in a two-dimensional sand reservoir model have been developed in order to study how to increase the efficiency of artificial replenishment of groundwater reservoir.

In [4], the authors simulated the process of groundwater filtration taking into account the nonuniform distribution and rarefaction of the aquifer based on insufficient data on the object and poor knowledge of their properties. The problem is solved by considering vertical stratification of an aquifer of equal thickness.

In [5], an exact solution was constructed on the influx of fluid into well-permeable formations. However, the study did not take into account the elastic regime in the low-permeable coffer dam.

Articles [6–7] are devoted to numerical modeling of water and salt transfer process in soil. For a comprehensive study, a mathematical model is proposed taking into account the soil pores colmatage with fine particles over time; changes in the coefficient of soil permeability, water loss and filtration coefficient; changes in the initial porosity and porosity of the settled mass, as well as an effective numerical algorithm based on the Samarsky-Fryazinov vector scheme with the second order of differential operators approximation to a finite-difference one. To derive a mathematical model of salt transfer, it is assumed that the pressure gradient in the channel is constant and equal to atmospheric pressure. The calculation results for the proposed algorithms are presented in the form of graphical objects.

In [8], a model is proposed that allows obtaining reliable information about the changes in the groundwater level and justifies the intensity of water reclamation of agro-landscapes, optimizes the agricultural drainage calculation and adjusts the water regime management in agricultural land.

Based on the data of complex studies carried out within the Ararat and Aparan intermountain basins, the issues related to the prevention of environmental consequences caused by large groundwater intakes are proposed in [9]. As a result of the data analysis of the mathematical modeling method, the problems of predicting the regime of groundwater levels changes were solved while maintaining a constant load acting on the existing water intakes.

In [10], a mathematical model was developed for predicting groundwater levels in two-layer formations. The authors consider a two-layer medium consisting of two layers: soil (with low conductivity) and water, as a mathematical model of a geofiltration process.

Statement of the problem

For mathematical modeling of monitoring and predicting the groundwater level and hydrochemical processes occurring in them, considering the interaction of external factors: evaporation and infiltration, the studied object is presented schematically in the form shown in Fig. 1. According to the results of hydrogeological conditions analysis, the territory with groundwater renewal (GWR) in the geofiltration respect should be considered as a vertical two-layer medium consisting of two aquifers (with a relatively close permeability value) separated by a poorly permeable layer.

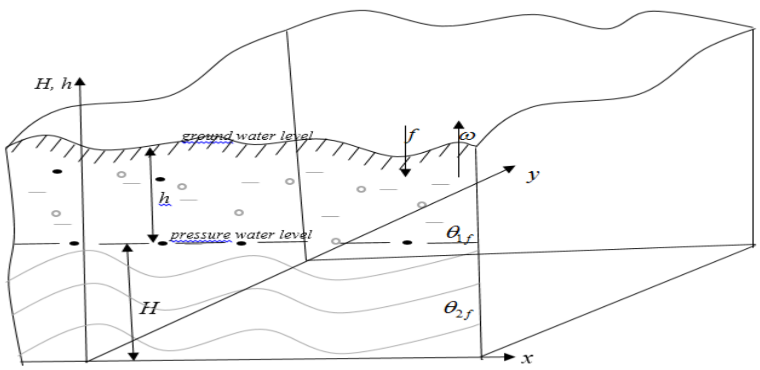

Fig 1.

The accepted conditions for predicting the groundwater level and the changes in salts content (ground and pressure aquifers) under filtration process give reason to present the mathematical model of the object in the form of a system of nonlinear partial differential equations:

$$\left.\begin{aligned} \mu_1 n_0 \frac{\partial h}{\partial t} &= \frac{\partial}{\partial x}(k_1 m \frac{\partial h}{\partial x}) + \frac{\partial}{\partial y}(k_1 m \frac{\partial h}{\partial y}) + f - \omega, \\ \mu_2 \frac{\partial H}{\partial t} &= \frac{\partial}{\partial x}\left(k_2 m \frac{\partial H}{\partial x}\right) + \frac{\partial}{\partial y}(k_2 m \frac{\partial H}{\partial y}) - \eta Q. \end{aligned}\right\} \quad (1)$$

where $h(x,y,t)$, $H(x,y,t)$ - are the ground and pressure water levels; μ_1, μ_2 are the water loss coefficients; m is the capacity of a separating layer; k_1, k_2 are the filtration coefficients of the upper and lower layers; Q is the flow rate; f is the external source; ω is the evaporation; n_0 is the active porosity of soil in the respective zones. η is the coefficient to reduce a model to dimensional form.

System (1) is solved under the following initial and boundary conditions:

$$h\big|_{t=0} = h_0, \quad H\big|_{t=0} = H_0, \tag{2}$$

$$\mu_1 m \frac{\partial h}{\partial x}\Big|_{x=0} = -(h - h_0), \quad \mu_1 m \frac{\partial h}{\partial x}\Big|_{x=L_x} = (h - h_0), \tag{3}$$

$$\mu_1 m \frac{\partial h}{\partial y}\Big|_{y=0} = -(h - h_0), \quad \mu_1 m \frac{\partial h}{\partial y}\Big|_{y=L_y} = (h - h_0), \tag{4}$$

$$\mu_2 m \frac{\partial H}{\partial x}\Big|_{x=0} = -(H - H_0), \quad \mu_2 m \frac{\partial H}{\partial x}\Big|_{x=L_x} = (H - H_0), \tag{5}$$

$$\mu_2 m \frac{\partial H}{\partial y}\Big|_{y=0} = -(H - H_0), \quad \mu_2 m \frac{\partial H}{\partial y}\Big|_{y=L_y} = (H - H_0), \tag{6}$$

$$H\big|_{y=m+0} = h\big|_{y=m-0}, \tag{7}$$

$$k_2 m \frac{\partial H}{\partial y}\Big|_{y=m+0} = k_1 m \frac{\partial h}{\partial y}\Big|_{y=m-0}. \tag{8}$$

where h_0, H_0 are the initial values of groundwater and pressure water levels.

To solve the problems (1) and (8), introduce the following dimensionless variables:

$$h^* = \frac{h}{h_0},\ H^* = \frac{H}{H_0},\ x^* = \frac{x}{L_x},\ y^* = \frac{y}{L_y},\ k_1^* = \frac{k_1}{(k_1)_0},\ \tau = \frac{(k_1)_0 m_0}{\mu_1 n_0 L_x^2} t,\ m^* = \frac{m}{m_0},\ k_2^* = \frac{k_2}{(k_2)_0},$$

Then the problem (1) - (8) is reduced to the form:

$$\left.\begin{aligned} \frac{\partial h^*}{\partial \tau} &= \frac{\partial}{\partial x^*}\left(k_1^* m^* \frac{\partial h^*}{\partial x^*}\right) + \frac{L_x^2}{L_y^2}\frac{\partial}{\partial y^*}\left(k_1^* m^* \frac{\partial h^*}{\partial y^*}\right) + \frac{L_x^2}{(k_1)_0 m_0 h_0}(f - \omega), \\ \frac{\partial H^*}{\partial \tau} &= \frac{\partial}{\partial x^*}\left(k_2^* m^* \frac{\partial H^*}{\partial x^*}\right) + \frac{L_x^2}{L_y^2}\frac{\partial}{\partial y^*}\left(k_2^* m^* \frac{\partial H^*}{\partial y^*}\right) - \frac{\mu_1 n_0 L_x^2}{\mu_2 (k_1)_0 m_0 H_0}\eta Q. \end{aligned}\right\} \tag{9}$$

Under boundary conditions:

$$\frac{\mu_1 m_0 h_0}{L_x} m^* \frac{\partial h^*}{\partial x^*}\Big|_{x^*=0} = -(h_0 h^* - h_0), \quad \frac{\mu_1 m_0 h_0}{L_x} m^* \frac{\partial h^*}{\partial x^*}\Big|_{x^*=1} = (h_0 h^* - h_0), \tag{10}$$

$$\frac{\mu_1 m_0 h_0}{L_y} m^* \frac{\partial h^*}{\partial y^*}\Big|_{y^*=0} = -(h_0 h^* - h_0), \quad \frac{\mu_1 m_0 h_0}{L_y} m^* \frac{\partial h^*}{\partial y^*}\Big|_{y^*=1} = (h_0 h^* - h_0), \tag{11}$$

$$\frac{\mu_2 m_0 H_0}{L_x} m^* \frac{\partial H^*}{\partial x^*}\Big|_{x^*=0} = -(H_0 H^* - H_0), \quad \frac{\mu_2 m_0 H_0}{L_x} m^* \frac{\partial H^*}{\partial x^*}\Big|_{x^*=1} = (H_0 H^* - H_0), \tag{12}$$

$$\frac{\mu_2 m_0 H_0}{L_y} m^* \frac{\partial H^*}{\partial y^*}\Big|_{y^*=0} = -(H_0 H^* - H_0), \quad \frac{\mu_2 m_0 H_0}{L_y} m^* \frac{\partial H^*}{\partial y^*}\Big|_{y^*=1} = (H_0 H^* - H_0), \tag{13}$$

$$H_0 H^*\Big|_{y^*=\frac{m_0 m^*+0}{L_y}} = h_0 h^*\Big|_{y^*=\frac{m_0 m^*-0}{L_y}}, \tag{14}$$

$$\frac{(k_2)_0 m_0 H_0}{L_y} k_2^* m^* \frac{\partial H^*}{\partial y^*}\Bigg|_{y^*=\frac{m_0 m^*+0}{L_y}} = \frac{(k_1)_0 m_0 h_0}{L_y} k_1^* m^* \frac{\partial h^*}{\partial y^*}\Bigg|_{y^*=\frac{m_0 m^*-0}{L_y}}. \tag{15}$$

Later, for simplicity, we will omit the "*" sign in the equations and problem (9) - (15) in dimensionless variables can be written as follows:

$$\left.\begin{aligned} \frac{\partial h}{\partial \tau} &= \frac{\partial}{\partial x}(k_1 m \frac{\partial h}{\partial x}) + \frac{L_x^2}{L_y^2}\frac{\partial}{\partial y}(k_1 m \frac{\partial h}{\partial y}) + \frac{L_x^2}{(k_1)_0 m_0 h_0}(f-\omega), \\ \frac{\partial H}{\partial \tau} &= \frac{\partial}{\partial x}\left(k_2 m \frac{\partial H}{\partial x}\right) + \frac{L_x^2}{L_y^2}\frac{\partial}{\partial y}(k_2 m \frac{\partial H}{\partial y}) - \frac{\mu_1 n_0 L_x^2}{\mu_2 (k_1)_0 m_0 H_0}\eta Q. \end{aligned}\right\} \tag{16}$$

Problem (16) has the form:

$$\left.\begin{aligned} \frac{\partial h}{\partial \tau} &= \frac{\partial}{\partial x}(k_1 m \frac{\partial h}{\partial x}) + \xi \frac{\partial}{\partial y}(k_1 m \frac{\partial h}{\partial y}) + \xi_1 (f-\omega), \\ \frac{\partial H}{\partial \tau} &= \frac{\partial}{\partial x}\left(k_2 m \frac{\partial H}{\partial x}\right) + \xi \frac{\partial}{\partial y}(k_2 m \frac{\partial H}{\partial y}) - \xi_2 \eta Q. \end{aligned}\right\} \tag{16*}$$

where $\xi = \frac{L_x^2}{L_y^2}$, $\xi_1 = \frac{L_x^2}{(k_1)_0 m_0 h_0}$, $\xi_2 = \frac{\mu_1 n_0 L_x^2}{\mu_2 (k_1)_0 m_0 H_0}$.

under boundary conditions:

$$\frac{\mu_1 m_0 h_0}{L_x} m \frac{\partial h}{\partial x}\Big|_{x=0} = -(h_0 h - h_0), \quad \frac{\mu_1 m_0 h_0}{L_x} m \frac{\partial h}{\partial x}\Big|_{x=1} = (h_0 h - h_0), \tag{17}$$

$$\frac{\mu_1 m_0 h_0}{L_y} m \frac{\partial h}{\partial y}\Big|_{y=0} = -(h_0 h - h_0), \quad \frac{\mu_1 m_0 h_0}{L_y} m \frac{\partial h}{\partial y}\Big|_{y=1} = (h_0 h - h_0), \tag{18}$$

$$\frac{\mu_2 m_0 H_0}{L_x} m \frac{\partial H}{\partial x}\Bigg|_{x=0} = -(H_0 H - H_0), \quad \frac{\mu_2 m_0 H_0}{L_x} m \frac{\partial H}{\partial x}\Bigg|_{x=1} = (H_0 H - H_0), \tag{19}$$

$$\frac{\mu_2 m_0 H_0}{L_y} m \frac{\partial H}{\partial y}\Bigg|_{y=0} = -(H_0 H - H_0), \quad \frac{\mu_2 m_0 H_0}{L_y} m \frac{\partial H}{\partial y}\Bigg|_{y=1} = (H_0 H - H_0), \tag{20}$$

$$H_0 H\Big|_{y=\frac{m_0 m+0}{L_y}} = h_0 h\Big|_{y=\frac{m_0 m-0}{L_y}}, \tag{21}$$

$$\frac{(k_2)_0 m_0 H_0}{L_y} k_2 m \frac{\partial H}{\partial y}\Bigg|_{y=\frac{m_0 m+0}{L_y}} = \frac{(k_1)_0 m_0 h_0}{L_y} k_1 m \frac{\partial h}{\partial y}\Bigg|_{y=\frac{m_0 m-0}{L_y}}. \tag{22}$$

3 Solution method

To solve problem (16*) - (22) the finite difference method [6, 7, 10] is used. For this, introduce a grid where T is the maximum time during which the process is studied for the domain $D=\{0\le x<L_x,\ 0\le y<L_y,\ 0\le t\le T\}$. To do so, the continuous domain of the problem solution is replaced by a grid one:

$$\omega_{\Delta x,\Delta y,\Delta\tau}=\{(x_i,y_j,t_n),\ x_i=i\,\Delta x;\ i=0,1,2,...I;\ y_j=j\,\Delta y;\ j=0,1,2,...J;\ t_n=n\Delta\tau;\ n=0,1,2,...,N\}$$

Next, we approximate equation (16 *) for the layer $n+\frac{1}{2}$ and use the implicit scheme on the grid $\omega_{\Delta x,\Delta y,\Delta\tau}$ in the form [6,7,10]:

$$\left.\begin{aligned}
&\frac{0.5\Delta\tau(k_1)_{i-0.5,j}m_{i-0.5,j}}{\Delta x^2}h_{i-1,j}^{n+\frac{1}{2}}-\frac{0.5\Delta\tau((k_1)_{i-0.5,j}m_{i-0.5,j}+(k_1)_{i+0.5,j}m_{i+0.5,j})+\Delta x^2}{\Delta x^2}h_{i,j}^{n+\frac{1}{2}}+\\
&\qquad+\frac{0.5\Delta\tau(k_1)_{i+0.5,j}m_{i+0.5,j}}{\Delta x^2}h_{i+1,j}^{n+\frac{1}{2}}=\\
&=-(h_{i,j}^n+0.5\Delta\tau\xi\frac{(k_1)_{i,j-0.5}m_{i,j-0.5}h_{i,j-1}^n-((k_1)_{i,j-0.5}m_{i,j-0.5}+(k_1)_{i,j+0.5}m_{i,j+0.5})h_{i,j}^n}{\Delta y^2}+\\
&\qquad+0.5\Delta\tau\xi\frac{(k_1)_{i,j+0.5}m_{i,j+0.5}h_{i,j+1}^n}{\Delta y^2}+0.5\Delta\tau\xi_1(f-\omega)),\\
&\frac{0.5\Delta\tau(k_2)_{i-0.5,j}m_{i-0.5,j}}{\Delta x^2}H_{i-1,j}^{n+\frac{1}{2}}-\frac{0.5\Delta\tau((k_2)_{i-0.5,j}m_{i-0.5,j}+(k_2)_{i+0.5,j}m_{i+0.5,j})+\Delta x^2}{\Delta x^2}H_{i,j}^{n+\frac{1}{2}}+\\
&\qquad+\frac{0.5\Delta\tau(k_2)_{i+0.5,j}m_{i+0.5,j}}{\Delta x^2}H_{i+1,j}^{n+\frac{1}{2}}=\\
&=-(H_{i,j}^n+0.5\Delta\tau\xi\frac{(k_2)_{i,j-0.5}m_{i,j-0.5}H_{i,j-1}^n-((k_2)_{i,j-0.5}m_{i,j-0.5}+(k_2)_{i,j+0.5}m_{i,j+0.5})H_{i,j}^n}{\Delta y^2}+\\
&\qquad+0.5\Delta\tau\xi\frac{(k_2)_{i,j+0.5}m_{i,j+0.5}H_{i,j+1}^n}{\Delta y^2}-0.5\Delta\tau\xi_2\eta Q).
\end{aligned}\right\}\quad(23)$$

After some transforms and grouping similar terms, the finite-difference system (23) is rewritten in the form:

$$a_{i,j}h_{i-1,j}^{n+\frac{1}{2}}-b_{i,j}h_{i,j}^{n+\frac{1}{2}}+c_{i,j}h_{i+1,j}^{n+\frac{1}{2}}=-d_{i,j}^n \quad (24)$$

$$\bar{a}_{i,j}H_{i-1,j}^{n+\frac{1}{2}}-\bar{b}_{i,j}H_{i,j}^{n+\frac{1}{2}}+\bar{c}_{i,j}H_{i+1,j}^{n+\frac{1}{2}}=-\bar{d}_{i,j}^n \quad (25)$$

here

$$a_{i,j}=\frac{0.5\Delta\tau(k_1)_{i-0.5,j}m_{i-0.5,j}}{\Delta x^2},\qquad b_{i,j}=\frac{0.5\Delta\tau((k_1)_{i-0.5,j}m_{i-0.5,j}+(k_1)_{i+0.5,j}m_{i+0.5,j})+\Delta x^2}{\Delta x^2},$$

$$c_{i,j}=\frac{0.5\Delta\tau(k_1)_{i+0.5,j}m_{i+0.5,j}}{\Delta x^2},$$

$$d_{i,j}^{n} = h_{i,j}^{n} + 0.5\Delta\tau\xi \frac{(k_1)_{i,j-0.5} m_{i,j-0.5} h_{i,j-1}^{n} - ((k_1)_{i,j-0.5} m_{i,j-0.5} + (k_1)_{i,j+0.5} m_{i,j+0.5}) h_{i,j}^{n}}{\Delta y^2} +$$

$$+ 0.5\Delta\tau\xi \frac{(k_1)_{i,j+0.5} m_{i,j+0.5} h_{i,j+1}^{n}}{\Delta y^2} + 0.5\Delta\tau\xi_1 (f - \omega),$$

$$\bar{a}_{i,j} = \frac{0.5\Delta\tau(k_2)_{i-0.5,j} m_{i-0.5,j}}{\Delta x^2}, \qquad \bar{b}_{i,j} = \frac{0.5\Delta\tau((k_2)_{i-0.5,j} m_{i-0.5,j} + (k_2)_{i+0.5,j} m_{i+0.5,j}) + \Delta x^2}{\Delta x^2},$$

$$\bar{c}_{i,j} = \frac{0.5\Delta\tau(k_2)_{i+0.5,j} m_{i+0.5,j}}{\Delta x^2},$$

$$\bar{d}_{i,j}^{n} = H_{i,j}^{n} + 0.5\Delta\tau\xi \frac{(k_2)_{i,j-0.5} m_{i,j-0.5} H_{i,j-1}^{n} - ((k_2)_{i,j-0.5} m_{i,j-0.5} + (k_2)_{i,j+0.5} m_{i,j+0.5}) H_{i,j}^{n}}{\Delta y^2} +$$

$$+ 0.5\Delta\tau\xi \frac{(k_2)_{i,j+0.5} m_{i,j+0.5} H_{i,j+1}^{n}}{\Delta y^2} - 0.5\Delta\tau\xi_2 \eta Q.$$

The resulting systems of equations (24) and (25) with respect to the sought for variables are solved by the sweep method, where the sweep coefficients are calculated using:

$$h_{i,j}^{n+\frac{1}{2}} = \alpha_{i+1,j} h_{i+1,j}^{n+\frac{1}{2}} + \beta_{i+1,j}^{n} \tag{27}$$

$$H_{i,j}^{n+\frac{1}{2}} = \bar{\alpha}_{i+1,j} H_{i+1,j}^{n+\frac{1}{2}} + \bar{\beta}_{i+1,j}^{n} \tag{28}$$

$\alpha_{i,j}$, $\beta_{i,j}^{n}$ and $\bar{\alpha}_{i,j}$, $\bar{\beta}_{i,j}^{n}$ are the sweep coefficients

$$\alpha_{i+1,j} = \frac{c_{i,j}}{b_{i,j} - a_{i,j}\alpha_{i,j}}, \quad \beta_{i+1,j}^{n} = \frac{d_{i,j}^{n} + a_{i,j}\beta_{i,j}^{n}}{b_{i,j} - a_{i,j}\alpha_{i,j}}, \quad \bar{\alpha}_{i+1,j} = \frac{\bar{c}_{i,j}}{\bar{b}_{i,j} - \bar{a}_{i,j}\bar{\alpha}_{i,j}}, \quad \bar{\beta}_{i+1,j}^{n} = \frac{\bar{d}_{i,j}^{n} + \bar{a}_{i,j}\bar{\beta}_{i,j}^{n}}{\bar{b}_{i,j} - \bar{a}_{i,j}\bar{\alpha}_{i,j}}.$$

Next, the boundary conditions (32) - (37) are approximated:

$$\frac{\mu_1 m_0 h_0}{L_x} m_{1,j} \frac{h_{0,j}^{n+\frac{1}{2}} - 4h_{1,j}^{n+\frac{1}{2}} + 3h_{2,j}^{n+\frac{1}{2}}}{2\Delta x} = -(h_0 h_{1,j}^{n+\frac{1}{2}} - h_0), \tag{29}$$

$$\frac{\mu_1 m_0 h_0}{L_x} m_{I,j} \frac{-3h_{I-1,j}^{n+\frac{1}{2}} + 4h_{I,j}^{n+\frac{1}{2}} - h_{I+1,j}^{n+\frac{1}{2}}}{2\Delta x} = (h_0 h_{I,j}^{n+\frac{1}{2}} - h_0), \tag{30}$$

$$\frac{\mu_1 m_0 h_0}{L_y} m_{i,1} \frac{h_{i,0}^{n+1} - 4h_{i,1}^{n+1} + 3h_{i,2}^{n+1}}{2\Delta y} = -(h_0 h_{i,1}^{n+1} - h_0), \tag{31}$$

$$\frac{\mu_1 m_0 h_0}{L_y} m_{i,J} \frac{-3h_{i,J-1}^{n+1} + 4h_{i,J}^{n+1} - h_{i,J+1}^{n+1}}{2\Delta y} = (h_0 h_{i,J}^{n+1} - h_0), \tag{32}$$

$$\frac{\mu_2 m_0 H_0}{L_x} m_{1,j} \frac{H_{0,j}^{n+\frac{1}{2}} - 4H_{1,j}^{n+\frac{1}{2}} + 3H_{2,j}^{n+\frac{1}{2}}}{2\Delta x} = -(H_0 H_{1,j}^{n+\frac{1}{2}} - H_0), \tag{33}$$

$$\frac{\mu_2 m_0 H_0}{L_x} m_{I,j} \frac{-3H_{I-1,j}^{n+\frac{1}{2}} + 4H_{I,j}^{n+\frac{1}{2}} - H_{I+1,j}^{n+\frac{1}{2}}}{2\Delta x} = (H_0 H_{I,j}^{n+\frac{1}{2}} - H_0), \tag{34}$$

$$\frac{\mu_2 m_0 H_0}{L_y} m_{i,1} \frac{H_{i,0}^{n+1} - 4H_{i,1}^{n+1} + 3H_{i,2}^{n+1}}{2\Delta y} = -(H_0 H_{i,1}^{n+1} - H_0), \tag{35}$$

$$\frac{\mu_1 m_0 H_0}{L_y} m_{i,J} \frac{-3H_{i,I-1}^{n+1} + 4H_{i,J}^{n+1} - H_{i,J+1}^{n+1}}{2\Delta y} = (H_0 H_{i,J}^{n+1} - H_0), \tag{36}$$

$$H_0 H_{i,J}^n = h_0 h_{i,J}^n \tag{37}$$

$$\frac{(k_2)_0 m_0 H_0}{L_y} (k_2)_{i,J} m_{i,J} \frac{-3H_{i,I-1}^n + 4H_{i,J}^n - H_{i,J+1}^n}{2\Delta y} = \frac{(k_1)_0 m_0 h_0}{L_y} (k_1)_{i,J} m_{i,J} \frac{-3h_{i,I-1}^n + 4h_{i,J}^n - h_{i,J+1}^n}{2\Delta y}. \tag{38}$$

If $i=1$, then equation (24) is transformed to equation (39), and as a result of simplification of equation (29), we obtain (40). If $i=0$, equation (27) is transformed to equation (41):

$$h_{2,j}^{n+\frac{1}{2}} = -\frac{a_{1,j}}{c_{1,j}} h_{0,j}^{n+\frac{1}{2}} + \frac{b_{1,j}}{c_{1,j}} h_{1,j}^{n+\frac{1}{2}} - \frac{d_{1,j}^n}{c_{1,j}} \tag{39}$$

$$h_{2,j}^{n+\frac{1}{2}} = -\frac{1}{3} h_{0,j}^{n+\frac{1}{2}} + \left(\frac{4}{3} - \frac{2\Delta x L_x h_0}{3\mu_1 m_0 h_0 m_{1,j}}\right) h_{1,j}^{n+\frac{1}{2}} + \frac{2\Delta x L_x h_0}{3\mu_1 m_0 h_0 m_{1,j}} \tag{40}$$

$$h_{0,j}^{n+\frac{1}{2}} = \alpha_{1,j} h_{1,j}^{n+\frac{1}{2}} + \beta_{1,j}^n \tag{41}$$

Comparing (39) - (41), we get $\alpha_{1,j}$ and $\beta_{1,j}^n$:

$$\alpha_{1,j} = \frac{3\mu_1 m_0 h_0 m_{1,j} b_{1,j} - 4\mu_1 m_0 h_0 m_{1,j} c_{1,j} + 2\Delta x L_x h_0 \mu_1 m_0 h_0 m_{1,j} c_{1,j}}{\mu_1 m_0 h_0 m_{1,j} (3a_{1,j} - c_{1,j})}, \quad \beta_{1,j}^n = -\frac{2\Delta x L_x h_0 c_{1,j}}{\mu_1 m_0 h_0 m_{1,j} (3a_{1,j} - c_{1,j})}.$$

At $i=I$ equation (24) takes the form (42), as a result of simplification of equation (30), we get (43), If $i=I-1$, equation (27) is transformed to equation (44):

$$h_{I+1,j}^{n+\frac{1}{2}} = -\frac{a_{I,j}}{c_{I,j}} h_{I-1,j}^{n+\frac{1}{2}} + \frac{b_{I,j}}{c_{I,j}} h_{I,j}^{n+\frac{1}{2}} - \frac{d_{I,j}^n}{c_{I,j}} \tag{42}$$

$$h_{I+1,j}^{n+\frac{1}{2}} = -3h_{I-1,j}^{n+\frac{1}{2}} + \frac{4\mu_1 m_0 h_0 m_{I,j} - 2\Delta x L_x h_0}{\mu_1 m_0 h_0 m_{I,j}} h_{I,j}^{n+\frac{1}{2}} + \frac{2\Delta x L_x h_0}{\mu_1 m_0 h_0 m_{I,j}} \tag{43}$$

$$h_{I-1,j}^{n+\frac{1}{2}} = \alpha_{I,j} h_{I,j}^{n+\frac{1}{2}} + \beta_{I,j}^n \tag{44}$$

Comparing (42) - (44), we get $h_{I,j}^{n+\frac{1}{2}}$:

$$h_{I,j}^{n+\frac{1}{2}} = \frac{\beta_{I,j}^n \mu_1 m_0 h_0 m_{I,j}(a_{I,j} - 3c_{I,j}) + 2\Delta x L_x h_0 c_{I,j} + d_{I,j}^n \mu_1 m_0 h_0 m_{I,j}}{b_{I,j}\mu_1 m_0 h_0 m_{I,j} - 4\mu_1 m_0 h_0 m_{I,j} c_{I,j} + 2\Delta x L_x h_0 c_{I,j} - \alpha_{I,j}\mu_1 m_0 h_0 m_{I,j}(a_{I,j} - 3c_{I,j})}$$

If $i=1$, then equation (25) is transformed to equation (45), and as a result of simplification of equation (33), we get (46). If $i=0$, then equation (28) is transformed to equation (47):

$$H_{2,j}^{n+\frac{1}{2}} = -\frac{\bar{a}_{1,j}}{\bar{c}_{1,j}} H_{0,j}^{n+\frac{1}{2}} + \frac{\bar{b}_{1,j}}{\bar{c}_{1,j}} H_{1,j}^{n+\frac{1}{2}} - \frac{\bar{d}_{1,j}^n}{\bar{c}_{1,j}} \tag{45}$$

$$H_{2,j}^{n+\frac{1}{2}} = -\frac{1}{3} H_{0,j}^{n+\frac{1}{2}} + \frac{4\mu_2 m_0 m_{1,j} - 2\Delta x L_x}{3\mu_2 m_0 m_{1,j}} H_{1,j}^{n+\frac{1}{2}} + \frac{2\Delta x L_x}{3\mu_2 m_0 m_{1,j}} \tag{46}$$

$$H_{0,j}^{n+\frac{1}{2}} = \bar{\alpha}_{1,j} H_{1,j}^{n+\frac{1}{2}} + \bar{\beta}_{1,j}^n \tag{47}$$

Comparing (45) - (47), we get $\alpha_{1,j}$ and $\beta_{1,j}^n$:

$$\bar{\alpha}_{1,j} = \frac{3(3\mu_2 m_0 m_{1,j}\bar{b}_{1,j} - 4\mu_2 m_0 m_{1,j}\bar{c}_{1,j} + 2\Delta x L_x \bar{c}_{1,j})}{3\bar{a}_{1,j} - \bar{c}_{1,j}}, \quad \bar{\beta}_{1,j}^n = -\frac{3(3\mu_2 m_0 m_{1,j}\bar{d}_{1,j}^n + 2\Delta x L_x \bar{c}_{1,j})}{3\bar{a}_{1,j} - \bar{c}_{1,j}}.$$

At $i=I$ equation (25) takes the form (48), as a result of simplification of equation (34) we obtain (49). If $i=I-1$, then equation (28) is transformed to equation (50):

$$H_{I+1,j}^{n+\frac{1}{2}} = -\frac{\bar{a}_{I,j}}{\bar{c}_{I,j}} H_{I-1,j}^{n+\frac{1}{2}} + \frac{\bar{b}_{I,j}}{\bar{c}_{I,j}} H_{I,j}^{n+\frac{1}{2}} - \frac{\bar{d}_{I,j}^n}{\bar{c}_{I,j}}, \tag{48}$$

$$H_{I+1,j}^{n+\frac{1}{2}} = -3H_{I-1,j}^{n+\frac{1}{2}} + \frac{4\mu_2 m_0 m_{I,j} - 2\Delta x L_x}{\mu_2 m_0 m_{I,j}} H_{I,j}^{n+\frac{1}{2}} + \frac{2\Delta x L_x}{\mu_2 m_0 m_{I,j}}. \tag{49}$$

$$H_{I-1,j}^{n+\frac{1}{2}} = \bar{\alpha}_{I,j} H_{I,j}^{n+\frac{1}{2}} + \bar{\beta}_{I,j}^n \tag{50}$$

Comparing (48) - (50), we get $H_{I,j}^{n+\frac{1}{2}}$:

$$H_{I,j}^{n+\frac{1}{2}} = \frac{2\Delta x L_x \bar{c}_{I,j} + \mu_2 m_0 m_{I,j}\bar{d}_{I,j}^n + \mu_2 m_0 m_{I,j}(\bar{a}_{I,j} - 3\bar{c}_{I,j})\bar{\beta}_{I,j}^n}{\mu_2 m_0 m_{I,j}\bar{b}_{I,j} - 4\mu_2 m_0 m_{I,j}\bar{c}_{I,j} + 2\Delta x L_x \bar{c}_{I,j} - \mu_2 m_0 m_{I,j}(\bar{a}_{I,j} - 3\bar{c}_{I,j})\bar{\alpha}_{I,j}}$$

Using the above algorithm, we find the values of $\bar{\bar{\alpha}}_{i,1}$, $\bar{\bar{\beta}}_{i,1}^n$, $h_{i,J}^{n+1}$, $\tilde{\bar{\alpha}}_{i,1}$, $\tilde{\bar{\beta}}_{i,1}^n$, $H_{i,J}^{n+1}$, on the layers $n+1$.

$$\bar{\bar{\alpha}}_{i,1} = \frac{3\mu_1 m_0 h_0 m_{i,1}\bar{\bar{b}}_{i,1} - 4\mu_1 m_0 h_0 m_{i,1}\bar{\bar{c}}_{i,1} + 2\Delta y L_y h_0 \bar{\bar{c}}_{i,1}}{\mu_1 m_0 h_0 m_{i,1}(3\bar{\bar{a}}_{i,1} - \bar{\bar{c}}_{i,1})}, \quad \bar{\bar{\beta}}_{i,1}^n = -\frac{3\mu_1 m_0 h_0 m_{i,1}\bar{\bar{d}}_{i,1}^n + 2\Delta y L_y h_0 \bar{\bar{c}}_{i,1}}{\mu_1 m_0 h_0 m_{i,1}(3\bar{\bar{a}}_{i,1} - \bar{\bar{c}}_{i,1})}.$$

$$h_{i,J}^{n+1} = \frac{\bar{\bar{\beta}}_{i,J}^n \mu_1 m_0 h_0 m_{i,J}(\bar{\bar{a}}_{i,J} - 3\bar{\bar{c}}_{i,J}) + 2\Delta y L_y h_0 \bar{\bar{c}}_{i,J} + \bar{\bar{d}}_{i,J}^n \mu_1 m_0 h_0 m_{i,J}}{\bar{\bar{b}}_{i,J}\mu_1 m_0 h_0 m_{i,J} - 4\mu_1 m_0 h_0 m_{i,J}\bar{\bar{c}}_{i,J} + 2\Delta y L_y h_0 \bar{\bar{c}}_{i,J} - \bar{\bar{\alpha}}_{i,J}\mu_1 m_0 h_0 m_{i,J}(\bar{\bar{a}}_{i,J} - 3\bar{\bar{c}}_{i,J})}$$

$$\tilde{\tilde{\alpha}}_{i,1} = \frac{3\mu_2 m_0 m_{i,1}\tilde{\tilde{b}}_{i,1} - 4\mu_2 m_0 m_{i,1}\tilde{\tilde{c}}_{i,1} - 2\Delta y L_y \tilde{\tilde{c}}_{i,1}}{\mu_2 m_0 m_{i,1}(3\tilde{\tilde{a}}_{i,1} - \tilde{\tilde{c}}_{i,1})}, \quad \tilde{\tilde{\beta}}^n_{i,1} = -\frac{2\Delta y L_y \tilde{\tilde{c}}_{i,1} + 3\mu_2 m_0 m_{i,1}\tilde{\tilde{d}}^n_{i,1}}{\mu_2 m_0 m_{i,1}(3\tilde{\tilde{a}}_{i,1} - \tilde{\tilde{c}}_{i,1})}.$$

$$H^{n+1}_{i,J} = \frac{2\Delta y L_y \tilde{\tilde{c}}_{i,J} + \mu_1 m_0 m_{i,J}\tilde{\tilde{d}}^n_{i,J} + \mu_1 m_0 m_{i,J}(\tilde{\tilde{a}}_{i,J} - 3\tilde{\tilde{c}}_{i,J})\tilde{\tilde{\beta}}^n_{i,J}}{\mu_1 m_0 m_{i,J}\tilde{\tilde{b}}_{i,J} - 4\mu_1 m_0 m_{i,J}\tilde{\tilde{c}}_{i,J} + 2\Delta y L_y \tilde{\tilde{c}}_{i,J} - \mu_1 m_0 m_{i,J}(\tilde{\tilde{a}}_{i,J} - 3\tilde{\tilde{c}}_{i,J})\tilde{\tilde{\alpha}}_{i,J}}$$

CONCLUSION

To study the process of changes in salt content and groundwater level over time, a mathematical model has been developed described by a system of nonlinear partial differential equations with corresponding initial and boundary conditions with account for external sources.

A conservative numerical algorithm has been developed for computer experiments. The developed mathematical tool can significantly reduce the bulk of field studies and minimize expensive and resource-intensive experimental work.

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