THE COORDINATE METHOD IN THE ANALOGY OF THE PROPERTIES SPACE OF A TRIANGLE

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ABSTRACT

It is known that the figure which formed by the successive connection of three points that do not lie on one straight line is called a triangle as well as the set of points of the plane that lie at the same distance from a given point is called a circle. They are called the easiestplanimetric figures.

KEYWORDS: *Coordinate Method, Analogy, Triangle, Planimetric Figures, Geometry, Element.*

INTRODUCTION

Planimetricissues for these figures and their combinations are quite extensively presented in school geometry course. As a sequence of the interest of students and the study of such problems, they will have the opportunity to better master the planimetry of geometry. However, in some cases, students do not have the skills to quickly master stereometric tasks. This is because they create information, concepts and representations about the conditions in which figures in space coexist with each other. Therefore, in order to create the concepts of space among students, it is necessary to jointly teach them stereometric figures with their analogies on the plane. The plane is a two-dimensional space, and upon transition to a three-dimensional space, the dimensions of some elements of the analogy of planimetric figures increase by one. Then, along with the fact that the analog in three-dimensional space of a straight line in a plane is a plane, the analog of a triangle in space is a tetrahedron, and the analog of a circle is a sphere. Therefore, the properties associated with combinations of triangles and circles are preserved in combinations of a tetrahedron and a sphere **[1].** For example, on any triangle, you can inscribe a circle whose center is at the intersection point of the bisectors. The spatial analogy is as follows: a sphere can be inscribed in any tetrahedron, the center of which will be located at the intersection point of the bisectoral plane **[2].** We can see that similar problems can be solved using the coordinate method. In many cases, knowledge of theory alone is not enough to solve geometric problems. To solve this issue, you need to have skill and hard work. This skill is realized starting with the simplest problems of questions and solving more and more complex problems. These include the spatial analogy of the properties of figures on the plane. There are many different ways to solve problems, and the conditions for their use depend on the nature of the problems. When explaining to students our current theorem and problems, the coordinate method will be acceptable. When using the coordinate method, it is advisable to carry out the following sequence: 1) translate it into the coordinate language in accordance with the content of the problem; 2) we choose the origin of coordinates that is suitable for usus(it is necessary to achieve that several coordinates of the points of the given figure are equal to zero); 3) explain the properties of the figure in the language of coordinates; 4) return the results to the geometric language; 5) come to the answer of the tasks.

1. Theorem of Menelaus. On the line BC, CA and AB, which contain the sides of the triangle ABC, points A_1, B_1, C_1 are taken, respectively, which should not coincide with the vertices of the triangle. In order for the points A_1, B_1, C_1 to lie on one straight line, it is necessary and sufficient that the equality.

$$\frac{\vec{AC_1}}{\vec{C_1B}}\cdot\frac{\vec{BA_1}}{\vec{A_1C}}\cdot\frac{\vec{CB_1}}{\vec{B_1A}} = -1 \qquad (1)$$

The Proof. To prove the theorem in the coordinate method, an affine frame $R=(A;\vec{AC};\vec{AB})$ was chosen (Fig. 1). Then the coordinates of the vertices of the triangle ABC with respect to this frame will be $A(0;0)$, $B(0;1)$, and $C(1;0)$.

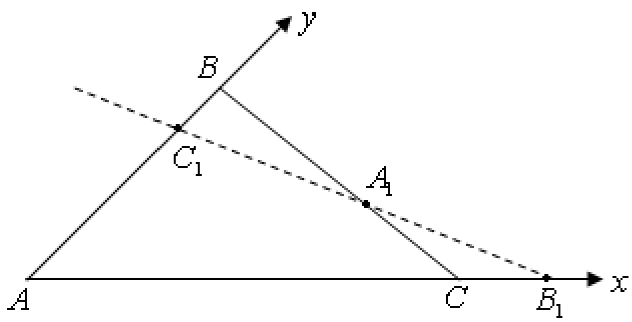

Figure 1.

Denote the points on the lines AB, BC, CA by C_1, A_1, B_1, respectively. These points must not coincide with the vertices of triangle ABC. If the number λ in the segment division formula is in this ratio, then designate λ_1 for point C_1 and for points A_1, $B_1$are marked as λ_2, λ_3,

$$\frac{\vec{AC_1}}{\vec{C_1B}}=\lambda_1,\quad \frac{\vec{BA_1}}{\vec{A_1C}}=\lambda_2,\quad \frac{\vec{CB_1}}{\vec{B_1A}}=\lambda_3.$$

Then equalities (1) are written in the form $\lambda_1\lambda_2\lambda_3=-1$.

Point C_1coordinates $(x_1;\, y_1)$ are calculated as:

$x_1 = \frac{0+\lambda_1 \cdot 0}{1+\lambda_1} = 0,\ y_1 = \frac{0+\lambda_1 \cdot 1}{1+\lambda_1} = \frac{\lambda_1}{1+\lambda_1}$.

It means $C_1(0; \frac{\lambda_1}{1+\lambda_1})$. Similarly, $A_1(\frac{\lambda_2}{1+\lambda_2}; \frac{1}{1+\lambda_2})$, $B_1(\frac{1}{1+\lambda_3}; 0)$. So, here *A*, *B*, *C* are not the same points, then $(1+\lambda_1)(1+\lambda_2)(1+\lambda_3) \neq 0$.

In order for the points A_1, B_1, C_1 to lie on the same line,

$$\begin{vmatrix} \frac{\lambda_2}{1+\lambda_2} & \frac{1}{1+\lambda_2} & 1 \\ \frac{1}{1+\lambda_3} & 0 & 1 \\ 0 & \frac{\lambda_1}{1+\lambda_1} & 1 \end{vmatrix} = 0$$

it is necessary and sufficient that this condition must be satisfied. On the other side,

$$\begin{vmatrix} \frac{\lambda_2}{1+\lambda_2} & \frac{1}{1+\lambda_2} & 1 \\ \frac{1}{1+\lambda_3} & 0 & 1 \\ 0 & \frac{\lambda_1}{1+\lambda_1} & 1 \end{vmatrix} = \frac{\lambda_1}{(1+\lambda_1)(1+\lambda_3)} - \frac{\lambda_1\lambda_2}{(1+\lambda_1)(1+\lambda_2)} - \frac{1}{(1+\lambda_2)(1+\lambda_3)} =$$

$$= -\frac{1}{(1+\lambda_1)(1+\lambda_2)(1+\lambda_3)}(\lambda_1\lambda_2\lambda_3 + 1).$$

Consequently, $\lambda_1\lambda_2\lambda_3 + 1 = 0$ and equivalent to equality (1) are necessary and sufficient condition for the lying points A_1, B_1, C_1 on the same line.

2. Analogy in space of Menelaus' theorem. On the lines *AB*, *BC*, *CD*, *DA* containing the edges of the tetrahedron *ABCD*, points C_1, A_1, B_1, D_1, respectively, are taken, which do not coincide with the vertices of the tetrahedron. In order for the points C_1, A_1, B_1, D_1 to lie on the same plane, it is necessary and sufficient that this condition be satisfied

$$\frac{\vec{AC_1}}{\vec{C_1B}} \cdot \frac{\vec{BA_1}}{\vec{A_1C}} \cdot \frac{\vec{CB_1}}{\vec{B_1D}} \cdot \frac{\vec{DD_1}}{\vec{D_1A}} = 1 \quad (2)$$

The Proof. For a given tetrahedron *ABCD*, it is convenient to choose the affine frame in the form $R=(A;\vec{AB};\vec{AC};\vec{AD})$. In this case, the coordinates of the vertices of the tetrahedron relative to the frame R have the form $A(0;0;0)$, $B(1;0;0)$, $C(0;1;0)$, $D(0;0;1)$ (2-fig.). Let the points C_1, A_1, B_1, D_1lie on the corresponding lines *AB*, *BC*, *CD*, *AD*, and do not coincide with the vertices of the tetrahedron. If

$$\frac{\vec{AC_1}}{\vec{C_1B}}=\lambda_1,\quad \frac{\vec{BA_1}}{\vec{A_1C}}=\lambda_2,\quad \frac{\vec{CB_1}}{\vec{B_1D}}=\lambda_3,\quad \frac{\vec{DD_1}}{\vec{D_1A}}=\lambda_4,$$

then equality (2) has the form $\lambda_1\lambda_2\lambda_3\lambda_4=1$ and $C_1(\frac{\lambda_1}{1+\lambda_1};0;0)$, $A_1(\frac{1}{1+\lambda_2};\frac{\lambda_2}{1+\lambda_2};0)$, $B_1(0;\frac{1}{1+\lambda_3};\frac{\lambda_3}{1+\lambda_3})$, $D_1(0;0;\frac{1}{1+\lambda_4})$.

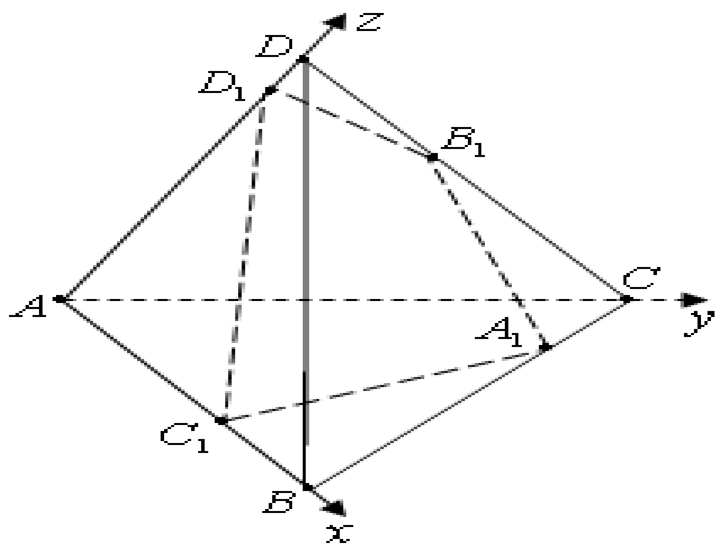

Figure 2.

For the points C_1, A_1, B_1, D_1 to lie on the same plane, it is necessary and sufficient that the determinant

$$\begin{vmatrix} \frac{1-\lambda_1\lambda_2}{(1+\lambda_1)(1+\lambda_2)} & \frac{\lambda_2}{1+\lambda_2} & 0 \\ \frac{-\lambda_1}{1+\lambda_1} & \frac{1}{1+\lambda_3} & \frac{\lambda_3}{1+\lambda_3} \\ \frac{-\lambda_1}{1+\lambda_1} & 0 & \frac{1}{1+\lambda_4} \end{vmatrix}$$

composed of vector coordinates

$$\vec{C_1A_1}\left\{\frac{1-\lambda_1\lambda_2}{(1+\lambda_1)(1+\lambda_2)};\frac{\lambda_2}{1+\lambda_2};0\right\},\ \vec{C_1B_1}\left\{\frac{-\lambda_1}{1+\lambda_1};\frac{1}{1+\lambda_3};\frac{\lambda_3}{1+\lambda_3}\right\},\ \vec{C_1D_1}\left\{\frac{-\lambda_1}{1+\lambda_1};0;\frac{1}{1+\lambda_4}\right\}$$

equals zero. On the other side,

$$\begin{vmatrix} \frac{1-\lambda_1\lambda_2}{(1+\lambda_1)(1+\lambda_2)} & \frac{\lambda_2}{1+\lambda_2} & 0 \\ \frac{-\lambda_1}{1+\lambda_1} & \frac{1}{1+\lambda_3} & \frac{\lambda_3}{1+\lambda_3} \\ \frac{-\lambda_1}{1+\lambda_1} & 0 & \frac{1}{1+\lambda_4} \end{vmatrix} = \frac{1-\lambda_1\lambda_2\lambda_3\lambda_4}{(1+\lambda_1)(1+\lambda_2)(1+\lambda_3)(1+\lambda_4)}.$$

Therefore, $\lambda_1\lambda_2\lambda_3\lambda_4 - 1 = 0$ and condition (2) are, in turn, a necessary and sufficient condition for the points C_1, A_1, B_1, $D_1$to lie on the same plane.

3. It is not always possible to describe the circumference of any polygon with more than three sides. For example, to describe the circumference of a quadrilateral, it is necessary and sufficient that the sum of its opposite angles be equal to 180^0. However, it is known from the course of planimetry that it is possible to describe a circle on any triangle, the center of which is the point of intersection of the perpendiculars drawn along the middle of the sides of this triangle. The spatial analogy of this property of a triangle can be given in the form of the following theorem.

Theorem. Nearly any tetrahedron, one can describe a sphere, the center of which is the intersection point of the planes drawn through the midpoints of the edges of the tetrahedron perpendicular to these edges.

The Proof. Let *ABCD* be any tetrahedron. We choose a rectangular Cartesian coordinate system, in which the vertex *A* of the tetrahedron is at the origin, the vertex *B* is in the abscissa axis, and the vertex *C* is on the *xy* plane (Figure 3). Then, the coordinate vertices of the given tetrahedron with respect to the coordinate system have the form $A(0;0;0)$, $B(b;0;0)$, $C(c_1;c_2;0)$, $D(d_1;d_2;\ d_3)$. Here the conditions $b\neq 0,\ c_2\neq 0,\ d_3\neq 0$ are supplied, since the vertices of the tetrahedron *ABCD* do not lie on the same plane.

If we designate the center of edges *AB*, *AC*and*AD*of the tetrahedron as M_1, M_2, M_3,respectively, then the Cartesian coordinates of these points according to the segment division formula in this ratio will be $M_1(\frac{b}{2};0;0)$, $M_2(\frac{c_1}{2};\frac{c_2}{2};0)$, $M_3(\frac{d_1}{2};\frac{d_2}{2};\frac{d_3}{2})$.

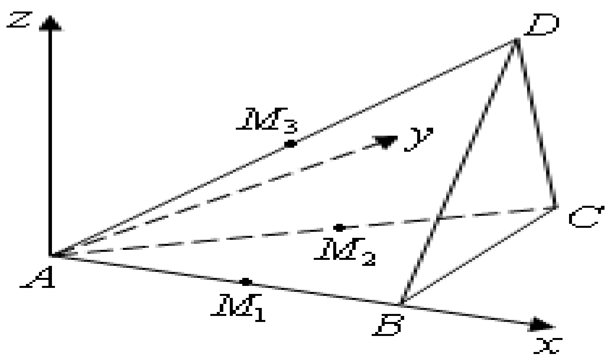

Figure 3.

The equation of the plane Π_1 passing through the point M_1 and perpendicular to the edge *AB* has the form:

$$(x-\frac{b}{2})\cdot b+(y-0)\cdot 0+(z-0)\cdot 0=0 \Rightarrow x-\frac{b}{2}=0.$$

Similarly to the plane passing through the point M_2 perpendicular to the edge *AC* and passing through the point M_3 perpendicular to the edge *AD*, respectively, denote Π_2 and Π_3 . Then, the equalities of these planes have the form:

$$\Pi_2: c_1x+c_2y-\frac{1}{2}(c_1^2+c_2^2)=0, \Pi_3: d_1x+d_2y+d_3y-\frac{1}{2}(d_1^2+d_2^2+d_3^2)=0.$$

The above planes Π_1, Π_2 and Π_3 intersect at one point $O(x_0; y_0; z_0)$. Since the main determinant

$$\begin{vmatrix} 1 & 0 & 0 \\ c_1 & c_2 & 0 \\ d_1 & d_2 & d_3 \end{vmatrix}$$

systems of equations

$$\begin{cases} x-\frac{b}{2}=0, \\ c_1x+c_2y-\frac{1}{2}(c_1^2+c_2^2)=0, \\ d_1x+d_2y+d_3y-\frac{1}{2}(d_1^2+d_2^2+d_3^2)=0. \end{cases}$$

is not equal to zero. Therefore, this system of equations has a unique solution $(x_0; y_0; z_0)$.

By definition of the plane $\Pi_1 \, \forall N \in \Pi_1 \Rightarrow AN = BN$ and $AO = BO$, since $O \text{O} \Pi_1$. Since this case is also true for planes Π_2 and Π_3, then $AO = CO, \; AO = DO$. This means that the sphere whose center is at the point $O(x_0; y_0; z_0)$ and whose radius is equal to the segment AO passes through all the vertices of the tetrahedron *ABCD*.

As a result of familiarizing students with the above tasks, we develop their ideas about the space.

REFERENCES:

1. Baymuratov XA., Tanirbergenov SA. Differences between planes and spatial figures. Scientific-methodical magazine "Continuous education", Tashkent; 2003;(5): 51-54.

2. Tanirbergenov SA. Properties of bisector of tetrahedron . Scientific Society, 2018;(2).