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CHOOSING THE BEST OPTION

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ABSTRACT

In this article, we analyzed several simple educational examples with small number of variables, in which you can get solutions using mathematics known to students in grades 7-10, proportions, properties of a linear function of no larger variable, a small enumeration of options. Methods for solving integer programming problems are also considered.

KEYWORDS: *Optimal, Best Option, Largest, Smallest, Integer Variable, Simplex Method, Basic Variable, Non-Basic Variable.*

INTRODUCTION

The problems of drawing up equations have, firstly to deal the translation of a condition from an ordinary language into a mathematical one, and secondly, to solve the resulting equations and inequalities. It is important to be able to solve such problems, sine in industrial and economic practice it is often necessary to summarize, recalculate and summarize various indicators, analyze the work of enterprises, etc. The result of this activity is an understanding of the current situation. After that, is it natural to take the next step-to draw up a plan-a program of further actions. Here, of course, many options formulation and solution of such problems are the subject of mathematical programming. In the process of solving problems with production content, we mean, first of all, instilling the ability to identify cause-and-effect relationships between economic factors and their mathematic interpretations. Was the soviet mathematician, academician L.V Kantorovich (1912-1986). In 1939 his book “Mathematical Methods of

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Organization and Planning of Production "was, published. In the introduction to this book, he efficiency of a shop, an enterprise and an entire industry. One way is various improvements in technology, that is, new devices in a separate machine, changing the technological process, so for less used, is trough improvements in production organization and planning. This includes, for example, such issues as the distribution of work between individual machine tools of an enterprise or mechanisms, the correct distribution of various types of raw materials, fuel, etc.

Many years have passed since then, and mathematical programming has become a goal of science, based on economics, mathematical methods and widely using computers. In real planning and control problems, one has to deal simultaneously with a very large number of variables. Well walk through a few simple examples with a small number of variables.

Example-1. Three alloys were brought to the laboratory. The first contains 40 percent copper and 60 percent nickel, the second 60 percent copper and 40 percent cobalt, the third 60 percent nickel. For the experiment, 1kg of a new alloy was needed, which would contain 40 percent cobalt and as little copper as possible (see table). How to make it?

Solution. Let's make a mathematical model of the problem. Take $x$kg

	+Cu	Ni	Co
1	40%	60%	-
2	60%	-	40%
3	-	40%	60%

of the first alloy, y kg of the second and z kg of the third.

By hypothesis $x+y+z=1$.

Cobalt in the new alloy will be $0{,}4y+0{,}6z$. Again, by condition $0{,}4y+0{,}6z=0{,}4$

Copper in the new alloy will be $0{,}4y+0{,}6z$. So, the model is built: it is required to find non-negative numbers x, y, z that satisfy the system of equations

$$x+y+z=1$$

$$0{,}4y+0{,}6z=0{,}4.$$

and for which the value of $f(x,y)=0{,}4x+0{,}6y$takes the smallest value.

Using the system of equations, we express x and z in terms of y and substitute their values into the smallest value of the resulting function of y (taking into account, of course, that all variables x, y and $z$must be non-negative).

From the second equation

$$z=\frac{2}{3}-\frac{2}{3}y.$$

Substitute this z-value into the first equation:

$$x=\frac{1}{3}-\frac{1}{3}y.$$

That's why

$$f(x,y) = \frac{2}{15} + \frac{7}{15}y.$$

Note that here we came across a linear function

$$f(x,y) = \frac{2}{15} + \frac{7}{15}y,$$

Which takes the smallest value at the smallest value $y = 0$. If we expressed $f(x,y)$ not in terms of y, but in terms of x or z, it would be more difficult to find out in which interval we should search for its smallest value. How best to choose which variable to reduce a function to is also an important task.

Example-2. It was decided to buy Christmas tree decorations for $100. Christmas decorations are sold in sets. A set of 20 toys costs $4, a set of 35 toys cost $6, and a set of 50 toys costs $9. Haw many and which nobles do you need to buy in order to buy the largest number of toys?

Solution. One toy in the first set costs$\$\frac{1}{5}$, in the second set-$\$\frac{6}{35}$, in the third set - $\$\frac{9}{50}$. Let's order these numbers:$\frac{6}{35} < \frac{9}{50} < \frac{1}{5}$.

So, the cheapest toys are in the second set, and the most expensive ones are in the first.

1	2	3
20	35	50
4$	5$	9$

1	2	3
1pc	1pc	1pc
$\frac{1}{5}\$$	$\frac{6}{35}\$$	$\frac{9}{30}\$$

To buy as many toys as possible for $100, of course, you need to buy more chear toys. At most, we can buy 16 sets at $6-and spend $96 on it. There remains $4, for which you can only buy the first set. In total, we will buy 16*35+20=580 toys in this way

This is most likely the best option. So, you needto buy 16 sets for $6 and 1 for $4. At the same time, 580 toys will be purchased.

We arrive at this answer with the help of such a natural reason: you can buy the more toys, the cheaper they are. Our reasoning is plausible,but, generally speaking, we have not looked through all the possible options. Therefore, we present a more rigorous solution.

Let x be the number of sets of type 1, $y - 2$types, $z - 3$ types. It is necessary to find such non-negative numbers x, y, z , so that conditions $4x + 6y + 9z \leq 100$ are fulfilled and the value $S = 20x + 35y + 50z$is the largest.

Since

$$4x + 6y + 9z = \frac{6}{35}S + \frac{4}{7}x + \frac{3}{7}z \geq \frac{6}{35}S,$$

It turn out that $\frac{6}{35}S \leq 100$, whence $S \leq 583\frac{1}{3}$. by 5, we have $S \leq 580$. For $x = 1, y = 16$, all conditions are satisfied and $S = 580$

This task belongs to integer programming, which is one of the most difficult areas of mathematical programming. A number of methods are used to solve linear integer programming problems. The simplest of these is the conventional linear programming method. If the components of the optimal solution turn out to be non-integer, they will be rounded to the nearest whole numbers. This method is used when a single unit of the population makes up a small part of the entire population. Otherwise, rounding can lead to a far from optimal target solution, therefore, specially developed methods are used.

Integer optimization methods can be divided into three main groups: 1)cut-off methods; 2)combinatorial methods; 3) approximate methods. Let's dwell on the clipping methods. The essence of pruning methods is that at first the problem is solved without the integer condition.

If the resulting plan is integer, the problem is solved. Otherwise, a new constraint is added to the constrains of the problem with the following properties:

1) it must be linear,

2) It must cut off the optimal non-integer plan found;

3) Must not cut off any integer plan.

Further, the problem is solved taking into account the new constraint. After that, if necessary, one more restriction is added, etc.

The linear integer programming problem is formulated as follows: find a solution $X = (x_1, x_2, \dots, x_n)$, such that the linear function

$$Z = \sum_{j=1}^{n} c_j x_j \quad (1)$$

takes the maximum or minimum value under the constraints

$$\sum_{i=1}^{n} a_{ij} x_j = b_i, i = 1,2, \dots, m \quad (2)$$

$$x_j \geq 0, j = 1,2, \dots, n \quad (3)$$

$$x_j - integers \quad (4)$$

Let the linear programming problem (1)-(4) have a finite optimum and at the last step it solution by the simplex method the following equations are obtained expressing the main variables $x_1, x_2, \dots, x_i, \dots, x_m$in terms of the minor variables $x_{m+1}, x_{m+2}, \dots, x_n$ of the optimal solution

$$\begin{cases} x_1 = \beta_1 - \alpha_{1m+1} x_{m+1} - \dots - \alpha_{1n} x_n, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ x_i = \beta_i - \alpha_{im+1} x_{m+1} - \dots - \alpha_{in} x_n, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ x_m = \beta_m - \alpha_{mm+1} x_{m+1} - \dots - \alpha_{mn} x_n \end{cases} \quad (5)$$

The optimal solution to problem (1)- (3) is

$X^* = (\beta_1, \beta_2, \ldots, \beta_i, \ldots, \beta_m, 0,0, \ldots, 0)$in which, for example,

β_i- is a noninteger component. In this case, one can prove that the in equality

$$\{\beta_i\} - \{\alpha_{im+1}\}x_{m+1} - \ldots - \{\alpha_m\}x_n \leq 0 \qquad (6)$$

formulated according to the 1-st equation of system (5), has all the properties of correct cutting. Inequality (6) contains the symbol { }, meaning the fractional part of the number.

To solve the integer linear programming problem (1)-(4) by the cut- of methods, the following algorithm is used:

1. Solve problem (1)-(3) using the simplex method without taking into account the integer condition. If all components of the optimal plan are integer, then it is also optimal for integer programming problems (1)-(4). If the first problem (1)-(3) is unsolvable, then the second problem (1)-(4) is also unsolvable.
2. If there are non-integral components among then components of the optimal solution, then select the component with the largest integral part and form the correct cutoff (6) using the corresponding equation of system(5).
3. Inequality (6) by introducing an additional nonnegative integer variable transform into an equivalent equation
$$\{\beta_i\} - \{\alpha_{im+1}\}x_{m+1} - \ldots - \{\alpha_{in}\}x_n + x_{n+1} \leq 0 \qquad (7)$$
4. Solve the resulting extended problem using the simplex method. If the found optimal plan is integer, then the integer programming problem (1)-(4) is solved. Otherwise, go back to algorithm 2.

Example-3. For the purchase of equipment for sorting grain, the farmer allocates 34 monetary units. The equipment must be located on an area not exceeding 60sq.m. the farmer can order two types of equipment: less powerful type A machines costing 3 monetary units, which requires a production area of 3 square meters (including aisles) and providing a productivity per shift of 2 tons of grain, and more powerful type B machines costing 4 monetary units, occupying an area of 5 square 4 meters and providing a productivity per shift of 3tons of high-quality grain.

An optimal equipment procurement plan is required ensure maximum overall productivity, provided the farmer can purchase no more than 8 type B machines.

Solution. Let us denote by $x_1, x_2$the number of machines of type A and B, respectively, by Z-the total productivity. Then the mathematical model of the problem will take the form:

$$Z = 2x_1 + 3x_2 \rightarrow max \qquad (1')$$

We the constraints:

$$\begin{cases} 3x_1 + 5x_2 \leq 60 & (1) \qquad (2') \\ 3x_1 + 4x_2 \leq 34 & (2) \\ x_2 \leq 8 & (3) \qquad (3') \end{cases}$$

$$x_1 \geq 0, x_2 \geq 0, x_1, x_2 - whole\ numbers \qquad (4')$$

Let us reduce the problem to the canonical form by introducing additional nonnegative variablesx_3, x_4, x_5. We get a system of restrictions:

$$\begin{cases} 3x_1 + 5x_2 + x_3 \qquad = 60 \\ 3x_1 + 4x_2 \qquad + x_4 \quad = 34 \\ \qquad x_2 \qquad\quad + x_5 = 8 \end{cases} \qquad (5')$$

$$x_j \geq 0, j = 1,2,\dots,5$$

Solving the problem using the using the simplex method

Step 1. Major variables x_3, x_4, x_5, minor variables x_1, x_2.

$$\begin{cases} x_3 = 60 - 3x_1 - 5x_2 \\ x_4 = 34 - 3x_1 - 4x_2 \\ x_5 = 8 \qquad - x_2 \end{cases}$$

$$Z = 2x_1 + 3x_2$$

First basic solution $X_1 = (0; 0; 60; 34; 8)$- admissible. The corresponding value of the linear function $Z_1 = 0$. We translate into the main variable x_2, which allows us to accept the system of restrictions, from the condition of the minimum of the corresponding relations

$$x_2 = min\left(\frac{60}{5}; \frac{34}{4}; \frac{8}{1}\right) = 8,$$

for $x_2 = 8$ in this equation $x_5 = 0$, and the variablex_5 goes to non-basic ones.

Step 2. Major variables x_2, x_3, x_4; minor variables x_1, x_5.

$$\begin{cases} x_2 = 8 - x_5 \\ x_3 = 20 - 3x_1 + 5x_2 \\ x_4 = 34 - 3x_1 + 4x_2 \end{cases}$$

$$Z = 24 + 2x_1 - 3x_5$$

$X_2 = (0; 8; 20; 2; 0)$;$Z_2 = 24$.

We translate into the main variable x_1,

$$x_1 = min\left(\infty; \frac{20}{3}; \frac{2}{3}\right) = \frac{2}{3},$$

and the non-main variable x_4.

Step 3. Major variables x_1, x_2, x_3; minor variables x_4, x_5.

After the transformations, we get

$$\begin{cases} x_1 = \frac{2}{3} - \frac{1}{3}x_4 + \frac{4}{3}x_5 \\ x_2 = 8 \qquad - x_5 \\ x_3 = 18 + x_4 + x_5 \end{cases}$$

$$Z = 25\frac{1}{3} - \frac{2}{3}x_4 - \frac{1}{3}x_5$$

The basic solutionx_3 is optimal for problem $(1') - (3')$, since there are no non-basic variables with positive coefficients in the expression of the linear function. However, solution x_3 does not satisfy the integer condition $(4')$. According to the first equation with variable x_1, which received a non- integer value in the optimal solution $\frac{2}{3}$, we compose an additional constraint (6):

$$\left\{\frac{2}{3}\right\} - \left\{\frac{1}{3}\right\}x_4 - \left\{\frac{4}{3}\right\}x_5 \leq 0$$

Since the fractional parts are

$$\left\{\frac{2}{3}\right\} = \left\{0 + \frac{2}{3}\right\} = \left\{\frac{2}{3}\right\},$$

$$\left\{\frac{1}{3}\right\} = \left\{0 + \frac{1}{3}\right\} = \left\{\frac{1}{3}\right\},$$

$$\left\{-\frac{4}{3}\right\} = \left\{-2 + \frac{2}{3}\right\} = \left\{\frac{2}{3}\right\},$$

we can write the last inequality in the form

$$\frac{2}{3} - \frac{1}{3}x_4 - \frac{2}{3}x_5 \leq 0 \qquad (6')$$

Introducing an additional integer variable $x_6 \geq 0$, we obtain the equation equivalent to inequality $(6')$

$$\frac{2}{3} - \frac{1}{3}x_4 - \frac{2}{3}x_5 + x_6 = 0 \quad (7')$$

<u>Step 4</u>. Basic variables $x_1, x_2, x_3, , x_5$;non-basic variables x_4, x_6

We get after the transformations:

$$\begin{cases} x_1 = 2 - x_4 + 2x_6 \\ x_2 = 7 + \frac{1}{2}x_4 - \frac{3}{2}x_6 \\ x_3 = 19 + \frac{1}{2}x_4 + \frac{3}{2}x_6 \\ x_5 = 1 - \frac{1}{2}x_4 + \frac{3}{2}x_6 \end{cases}$$

$$Z = 25 - \frac{1}{2}x_4 - \frac{1}{2}x_6$$

$$x_5 = (2; 7; 19; 0; 1; 0)$$

$$Z_5 = 25$$

So, $Z_{max} = 25$ for the optimal integer solution $X^* = x_5 = (2; 7; 19; 0; 1; 0)$, i.e. the maximum productivity of 25 tons of high-quality grain per shift can be obtained by purchasing 2 machines of type *A* and 7 machines of type B, while the unoccupied area of the premises will be 19sq.m, cash balances from the allocated funds are equal to 0, in the reserve for purchase-1 car of type B.

CONCLUSIONS

In our opinion, it useful for students of all faculties of an economic university to get acquainted with the methods of mathematical programming, the creation of which is associated with the urgent needs of planning and organizing production.

Deepening of economic ties between different branches of the National economy, an increase in the scale production not allow you to do without quantitative methods of economic calculations and usage of modern computers.

Methods of mathematical programming make it possible to allocate limited resources in the most rational way: whether it is the problem of the best use of limited production resources for the release of a certain set of products, the so-called production planning problem of the most efficient use of vehicles for the transportation of a given volume of products, a transport problem. At the same time, linear programming allows you to obtain such a distribution accurately, and not by eye.

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