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ON THE UNIQUENESS OF THE SOLUTION OF A TWO-POINT SECOND BOUNDARY VALUE PROBLEM FOR A SECOND-ORDER SIMPLE DIFFERENTIAL EQUATION SOLVED BY THE BERNOULLI EQUATION

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ABSTRACT

This article examines the uniqueness of the solution of the boundary problem for the second regular ordinary differential equation, which is solved in the Bernoulli equation. The uniqueness of the issue is proved by the principle of extremes.

KEYWORDS: *Ordinary Differential Equation, Unity Of A Solution, Availability Of A Solution.*

INTRODUCTION

Problem statement

$$y''+P_1(x)y'+P_2(x)y'^2+P_3(x)y'^3=P(x), \qquad x\in\left[x_0;x_1\right] \qquad (1)$$

equation and

$$y'\left(x_0\right)=y_0,\ \ y\left(x_1\right)=y_1 \qquad (2)$$

find the function $y(x)$ that satisfies the boundary conditions. Here $P(x), P_1(x), P_2(x), P_3(x)$ - given continuous functions.

Theorem. $p_1(x)-$ (1) is a special solution of equation (2) that does not satisfy the boundary condition, $P_2(x)+3P_3(x)p_1(x)=0$ and $P_1(x)+2P_2(x)p_1(x)+3P_3(x)p_1^2(x)=Q_1(x)$, $y_0-p_1(x_0)\neq 0$, $Q_1(x)\neq 0$, $p_1'(x)+P_1(x)p_1(x)+P_2(x)p_1^2(x)+P_3(x)p_1^3(x)=P(x)$ if the conditions are satisfied, then the problem S_3 has a unique solution.

Proof. Using the notation $y'=p(x)$ in Equation (1), we construct the following equation $p'(x)+P_1(x)p(x)+P_2(x)p^2(x)+P_3(x)p^3(x)=P(x)$ (3)

and condition $p(x_0)=y_0$ (4) from the boundary conditions (2).

The resulting {(3), (4)} is in the new case

$$p(x)=p_1(x)+z(x), \quad (5)$$

by performing the substitution and after some elementary simplification we form the

$$z'(x)+\left(P_1(x)+2P_2(x)p_1(x)+3P_3(x)p_1^2(x)\right)\cdot z+\left(P_2(x)+3P_3(x)p_1(x)\right)\cdot z^2+$$

$$+P_3(x)z^3(x)=P(x)-\left[p_1'(x)+P_1(x)p_1(x)+P_2(x)p_1^2(x)+P_3(x)p_1^3(x)\right]$$

equation. If we apply the conditions of the theorem to this equation

$$z'(x)+Q_1(x)z(x)=-P_3(x)z^3(x) \quad (6)$$

Based on the substitution of the Bernoulli equation in the form Sx and (4) from the boundary condition (5)

$$z(x_0)=y_0-p_1(x_0) \quad (7)$$

we create the condition. (6) in equation

$$t(x)=\frac{1}{z^2(x)} \quad (8)$$

by carrying out replacement,

$$t'(x)-2Q_1(x)t(x)=2P_3(x) \quad (9)$$

while the equation and (7) condition

$$t(x_0)=\frac{1}{\left[y_0-p_1(x_0)\right]^2} \quad (10)$$

we form the condition. As a result, we come to the new {(9), (10)} issue.

Let's assume, {(9),(10)} let the issue have t1 as well as t2 solutions. In that case

$$t(x)=t_1(x)-t_2(x) \quad (11)$$

function

$$t'(x)-2Q_1(x)t(x)=0\ ,\ x\in[x_0;x_1], \quad (9')$$

$$t(0)=0 \quad (10')$$

will be the solution to a problem.

Suppose that the problem {(9'),(10')} has a solution $t(x)\not\equiv 0,\ x\in[x_0;x_1]$.

Since the function $t(x)$ is defined and continuous in the segment $[x_0;x_1]$, it reaches a positive maximum (negative minimum) value at some point $x'\in(x_0;x_1]$ of this segment according to Weierstrass's theorem 2.

We assume that the function $t(x)$ should reach a positive maximum (minus minimum) value $(x_0,x_1]$ in half the range. Assuming that $t(x_0')>0\ \ (<0)$ is a positive maximum (minus minimum) value, then $t'(x')=0$ equality, as well as $t'(x)-2Q_1(x)t(x)>0$ inequality is executed.

This is contrary to $(9')$. Hence, $t(x)$ the function is (basically) at $\forall x'\in(x_0,x_1]$ ($(9')$ basically)

$$t(x)\equiv 0,\qquad \forall x'\in(x_0,x_1] \quad (12).$$

Based on this (11) $t_1(x)=t_2(x)$. And it turns out that the solution of the issue {(9),(10)} is no more than one. All in all, {(9),(10)} the issue is that if he has a solution in the cut $[x_0;x_1]$, It is the only one. {(9), (10)} since the solution of the issue is unique {(1), (2)} the solution of the issue is also unique. Because, {(9),(10)} the issue {(1), (2)} is an equivalent issue.

Availability of problem solutions. (9) Using the Bernoulli method to find the general solution of equation

$$t(x)=u(x)\cdot v(x) \quad (13)$$

apparently looking for. Substituting (13) into (9)

$$u'(x)\cdot v(x)+u(x)\cdot\left[v'(x)-2Q_1(x)\cdot v(x)\right]=2P_3(x)$$

creating equality, hence

$$v(x)=v(x_0)e^{-2\int_{x_0}^{x}Q_1(s)ds},$$

$$u(x)=u(x_0)-\frac{2}{v(x_0)}\cdot\int_{x_0}^{x}P_3(s)e^{2\int_{x_0}^{s}Q_1(\tau)d\tau}ds$$

find the functions and solve the general solution of equation (9) based on (13)

$$t(x)=\left[u(x_0)v(x_0)-2\cdot\int_{x_0}^{x}P_3(s)e^{2\int_{x_0}^{s}Q_1(\tau)d\tau}ds\right]\cdot e^{-2\int_{x_0}^{x}Q_1(s)ds},$$

$$t(x)=\left[t(x_0)-2\cdot\int_{x_0}^{x}P_3(s)e^{2\int_{x_0}^{s}Q_1(\tau)d\tau}ds\right]\cdot e^{-2\int_{x_0}^{x}Q_1(s)ds}$$

in the view. Subordinate this solution to the condition (10) Rx

$$z(x)=\pm\left\{\frac{1}{\left[y_1-p_1(x_0)\right]^2}-2\cdot\int_{x_0}^{x}P_3(s)e^{2\int_{x_0}^{s}Q_1(\tau)d\tau}ds\right\}^{-\frac{1}{2}}\cdot e^{\int_{x_0}^{x}Q_1(s)ds}$$

find the function. Going back to the substitution (5) above, we find the conditional solution of equation (4) in the form

$$p(x)=p_1(x)+\left\{\frac{1}{\left[y_1-p_1(x_0)\right]^2}-2\cdot\int_{x_0}^{x}P_3(s)e^{2\int_{x_0}^{s}Q_1(\tau)d\tau}ds\right\}^{-\frac{1}{2}}\cdot e^{\int_{x_0}^{x}Q_1(s)ds},$$

and finally, we find the conditional solution of equation (1) in the form $y'=p(x)$ using the notation

$$p(x)=p_1(x)+\left\{\frac{1}{\left[y_1-p_1(x_0)\right]^2}-2\cdot\int_{x_0}^{x}P_3(s)e^{2\int_{x_0}^{s}Q_1(\tau)d\tau}ds\right\}^{-\frac{1}{2}}\cdot e^{\int_{x_0}^{x}Q_1(s)ds}.$$

The theorem is fully proved.

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