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ABSTRACT

The article examines the turbulent flow of a fluid flow in a rotating pipe, which is called the Poiseuille-Couette-Taylor flow. The main approaches to the numerical simulation of turbulent flows in the annular region between rotating cylinders are considered. The calculated results are obtained, which correlate with the known experimental results. On the basis of a comparative analysis, the most suitable differential turbulence model is proposed for calculating the conjugate problems of hydrodynamics and heat transfer in a Poiseuille-Couette-Taylor flow.

KEYWORDS: *Mathematical Model Of Turbulence Based On The Dynamics Of Two Fluids,* SSG / LRR-RSM-W2012 Model, Swirling Flow.

INTRODUCTION

At present, the most effective approach to the study of turbulence is an analytical one, based on the initial premise that the Navier - Stokes system of equations describing the characteristics of an instantaneous fluid flow is acceptable for the mathematical description of turbulent flows. The main tools for calculating turbulent flows are numerical methods, the widespread use of which has become possible due to the rapid improvement of computer technology. However, despite the keen interest, until now there is no universal approach to the calculation of turbulent flows that would adequately reflect some aspects of these flows, which are manifested in various special cases.

The most accessible is the use of various turbulence models in combination with the Reynolds Averaged Navier-Stokes Equations (RANS). This approach to numerical modeling is less demanding on computational resources and therefore more accessible, and also does not have a pronounced limitation on the degree of turbulence of the flows under consideration. However, its application requires special attention to the verification of the obtained solution. All turbulence models used in the framework of the use of RANS equations contain empirical dependences and



ISSN: 2249-7137 Vol. 11, Issue 9, September 2021 Impact Factor: SJIF 2021 = 7.492

coefficients calibrated for a certain kind of flows. Therefore, none of these models provides the most adequate results in all cases. Moreover, it is known [1,2] that simple algebraic models, within the framework of their applicability, can provide results no worse than more complex differential models. In addition, it should be borne in mind that this or that turbulence model can adequately reflect some features or patterns of the considered turbulent flow, and not reflect or distort others. In this case, the success of the calculation depends on which physical features are predominant in this particular case and to what extent.

Turbulence modeling currently used in aerodynamics is usually based on two-equation models using a linear relationship between Reynolds stress and mean strain rate tensors. This relationship is known as the generalized Boussinesq model. This can be overly restrictive in complex tasks typical of high lift aerodynamics, because many different flow phenomena can be present in one task. Therefore, it is necessary to look for turbulence modeling with a wider range of applicability than the Boussinesq model. Reynolds Differential Voltage Modeling (RSM), in which the simulated transfer equation is solved for each stress component, is in principle a more general class of models with a wider range of applicability. However, RSM is considered an overly complex approach to industrial design of high lift aerodynamics. On the other hand, twoequation models can be extended to a wider range of applicability by developing more complex nonlinear relationships between stress tensor and mean velocity gradient and turbulent scales. These relationships are commonly referred to as constitutive models. One might think that twoequation turbulence models consist of two more or less separate parts: a scale-determining model, which provides scalar information about turbulence, and a constitutive model, which defines the Reynolds stress tensor. Explicit Algebraic Reynolds Stress Models (EARSM) represent an interesting and promising subset of nonlinear constitutive models. In this approach, part of the description of higher-order physical processes at the RSM level is transferred to the modeling level with two equations. The EARSM approach is considered a suitable type of constitutive modeling for the present purposes.

Statement of the problem

The physical formulation of the problem is shown in Figure 1. As can be seen from the figure, a laminar non-swirling flow enters the rotating pipe, and the flow at the outlet is completely turbulent and swirling. Therefore, a sufficiently long pipe is considered for, i.e. the length is substantially greater than its diameter. The considered flow is characterized by the Reynolds number, which is determined by the average flow rate and the radius of the pipe.

In the case of rotation, the rotation parameter $N = \frac{\pi D^2 \Omega R}{4Q}$ is also entered, where ρ is the

density of the liquid, Q is the volumetric flow rate, μ is the dynamic viscosity, and R is the radius of the pipe.





Fig. 1: Swirling flow through a rotating pipe.

Mathematical modeling of the problem

For the numerical simulation of the turbulent flow of an incompressible fluid, the Reynolds equations were used [3]:

$$\begin{cases} \frac{\partial \overline{U}_{i}}{\partial x_{i}} = 0, \\ \frac{\partial \overline{U}_{i}}{\partial t} + \overline{U}_{j} \frac{\partial \overline{U}_{i}}{\partial x_{i}} + \frac{1}{\rho} \frac{\partial \overline{p}}{\partial x_{i}} = \frac{\partial}{\partial x_{i}} \left[\nu \left(\frac{\partial \overline{U}_{i}}{\partial x_{j}} + \frac{\partial \overline{U}_{j}}{\partial x_{i}} \right) \right] + \frac{\partial (-\overline{u'_{j} u'_{i}})}{\partial x_{i}}. \end{cases}$$
(1)

The system of Navier-Stokes equations averaged over Reynolds (1) is not closed. For closure in methods, nonlinear turbulence approaches are used.

The mixed SSG/LRR second moment Reynolds stress model is a nonlinear RANS turbulence model that uses the omega equation for the length scale equation. Reynolds stress models with full second moment are very different from simpler linear or nonlinear single equation models, as the latter use a constitutive relation giving the Reynolds stresses τ_{ii} in terms of other tensors

through some assumed relation (such as Boussinesq's hypothesis).On the other hand, full second moment Reynolds stress models calculate each of the 6 Reynolds stresses directly (the Reynolds stress tensor is symmetric, so there are 6 independent terms). Each Reynolds stress has its own transfer equation. There is also a seventh transport equation for the scale variable. The complete Reynolds stress model SSG / LRR-omega (SSG / LRR-RSM-w2012) and one length scale equation are:



ISSN: 2249-7137 Vol. 11, Issue 9, September 2021 Impact Factor: SJIF 2021 = 7.492

$$\left(\frac{\partial R_{ij}}{\partial t} + \frac{\partial \overline{U}_{k} R_{ij}}{\partial x_{k}} = P_{ij} + \Pi_{ij} - \varepsilon_{ij} + D_{ij}, \\
\left(\frac{\partial \omega}{\partial t} + \frac{\partial \overline{U}_{k} \omega}{\partial x_{k}} = \frac{a_{\omega} \omega}{k} \frac{P_{kk}}{2} - B_{\omega} \omega^{2} + \frac{\partial}{\partial x_{k}} \left(\left(\mu + \sigma_{\omega} \frac{k}{\omega}\right) \frac{\partial \omega}{\partial x_{k}} \right) + \sigma_{d} \frac{1}{\omega} \frac{\partial k}{\partial x_{j}} \frac{\partial \omega}{\partial x_{j}}.$$
(4)

Here, $\rho R_{ij} = -\tau_{ij} = \rho \overline{u_i u_j}$, P_{ij} - generation of Reynolds stresses, D_{ij} - diffusion, ε_{ij} - dissipation, Π_{ij} - term of pressure redistribution. The remaining values of the initial and boundary conditions are presented in [1,2].

Malikov's new two-fluid turbulence model is presented in [4], which has the form
$$\begin{cases}
\frac{\partial \overline{U}_{i}}{\partial t} + \overline{U}_{j} \frac{\partial \overline{U}_{i}}{\partial x_{j}} = -\frac{1}{\rho} \frac{\partial \overline{p}_{i}}{\partial x_{i}} + \frac{\partial}{\partial x_{j}} \left[v \left(\frac{\partial \overline{U}_{i}}{\partial x_{j}} + \frac{\partial \overline{U}_{j}}{\partial x_{i}} \right) - u_{j} u_{i} \right], \\
\frac{\partial u_{i}}{\partial t} + \overline{U}_{j} \frac{\partial u_{i}}{\partial x_{j}} = -u_{j} \frac{\partial \overline{U}_{i}}{\partial x_{j}} + \frac{\partial}{\partial x_{j}} \left[v_{ij} \left(\frac{\partial u_{i}}{\partial x_{j}} + \frac{\partial u_{j}}{\partial x_{i}} \right) \right] + \frac{F_{fi}}{\rho} + \frac{F_{\perp i}}{\rho}, \\
v_{ij} = 3v + 2 \left| \frac{u_{i} u_{j}}{def(\overline{U})} \right|, \quad i \neq j, \quad v_{ii} = 3v + \frac{2}{div(vv)} \left| \frac{u_{k} u_{k}}{def(\overline{U})} \right| \frac{\partial \rho u_{k}}{\partial x_{k}}, \quad F_{f} = -\rho K_{f} u, \\
\frac{\partial \overline{U} p_{i}}{\partial t} + \overline{U}_{j} \frac{\partial \overline{U} p_{i}}{\partial x_{j}} = k_{m} (\overline{U}_{i} - \overline{U} p_{i}), \quad F_{\perp} = \rho C_{s} rot \overline{U} \times u, \quad \frac{\partial \overline{U}_{j}}{\partial x_{j}} = 0.
\end{cases}$$
(5)

Here, u_i –respectively axial, radial and tangential relative velocities, $C_s = 1$ Seffman's power factor, v_{ij} - kinematic molar viscosity, $def\overline{U}$ - deformation of the average flow rate. The last equation is the kinematic equations for the solid phase.

Results

In fig. 2. Numerical results of the nonlinear SSG / LRR-RSM model and two fluid models for longitudinal velocity are presented. The results of nonlinear turbulence approaches can be said to qualitatively describe the longitudinal velocity, while the two-fluid model describes quantitatively.



Fig. 2. Axial velocity profile in a rotating tube





In fig. 3. numerical results of both models for the tangential flow velocity are presented.

Fig.3. Tangential Velocity Profile in a Rotating Tube

It can be seen from this figure that the nonlinear model SSG / LRR-RSM is not even qualitatively able to describe the tangential velocity, which confirms the above statements. As for the new model, we can observe good agreement with experimental data..

CONCLUSION

The model demonstrated simplicity for numerical implementation and good robustness. The new model is economical in terms of counting time. For example, in comparison with the SSG / LRR-RSM-w2012 model, the model allows integration with a time step of 20 times more.

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