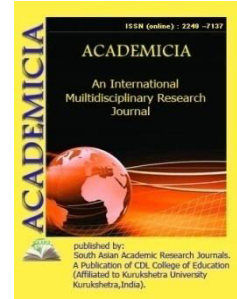




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A BRIEF DESCRIPTION ON THE FUNCTIONS OF TRIGONOMETRY

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ABSTRACT

There are a variety of mathematical ideas that are often utilized in real-world applications. Trigonometry is one such idea that is now widely utilized in a variety of tasks, including the construction of dams, bridges, and buildings. When it comes to solving real-world issues, trigonometric calculations have shown to be dependable and helpful. Working on it is both simple and enjoyable. In general, students find it difficult to grasp its ideas at first, but once they do, solving trigonometric problems becomes very interesting and enjoyable. Angles and sides are the focus of trigonometry. All trigonometric calculations and formulas are based entirely on triangle angles and sides. There are a number of trigonometric identities that may be used to assist solve trigonometric issues. This article has covered all of the ideas related to trigonometry, including identities, formulas, and graphical representations of trigonometric functions. This article describes how trigonometry is used in real-life situations. It discusses the future of trigonometry and the need of pupils fully understanding trigonometric principles.

KEYWORDS: *Angle, Mathematics, Side, Sine, Trigonometry.*

1. INTRODUCTION

Mathematics is the subject that students believe to be the most difficult. It's all about numbers in mathematics. The numbers 0 to 9 are the foundation of all mathematics. Math is entirely dependent on computations and logic[1]. Mathematicians employ a variety of theorems and rules to solve a variety of issues. In this article, a few of these theorems, rules, and ideas are briefly described.

1.1 Trigonometry:

Angles are the foundation of trigonometry. Cosecant, tangent, cosine, secant, sine, and cotangent are the six major functions in it. These may be expressed as cot, cos, cosec, tan, sec, and sin in abbreviated form, respectively[2], [3]. The inverse relationship between sin and cosec may be expressed as $\sin=1/\text{cosec}$ or vice versa. In the same way, cos and sec have an inverse relationship that may be expressed as $\cos =1/\text{sec}$ or vice versa. Tan and cot have the same inverse relationship, which is expressed as $\tan=1/\text{cot}$ or vice versa. Trigonometry is entirely dependent on the use of right-angled triangles to accomplish different functions. All of these elements are needed to calculate the sides and angles of triangles. The sign “°” is used to indicate angles. There are a number of formulas that link these elements to triangle sides and angles.

1.2 Trigonometric Functions:

Sin,cos,sec,cosine,tan and cot are six important functions in trigonometry which form base of trigonometry.

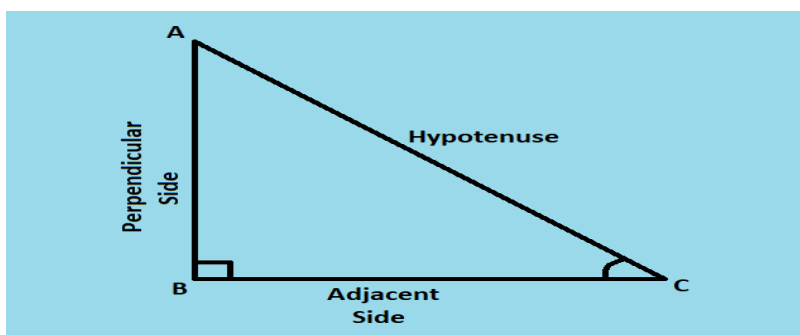


Figure 1: Triangle representing all three sides with respect to angle θ [4].

Figure 1 shows a triangle which represents all three sides with respect to angle θ .

$\sin \theta = \text{side opposite to angle}/\text{Hypotenuse of triangle}$

$\cos \theta = \text{side adjacent to angle}/\text{Hypotenuse of triangle}$

$\tan \theta = \text{side opposite to angle} / \text{side adjacent to angle}$

$\text{Cosec } \theta = \text{Hypotenuse of triangle}/\text{side opposite to angle}$

$\text{Sec } \theta = \text{Hypotenuse of triangle} / \text{side adjacent to angle}$

$\text{Cot } \theta = \text{Base}/\text{Perpendicular}$

There are various formulae which establish a relation between these components and can be used to find out solution of trigonometric problems[5]. These are called as Pythagorean identities. These are:

$$\cot^2\theta + 1 = \operatorname{cosec}^2\theta$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan 2\theta = 2 \tan \theta / (1 - \tan^2\theta)$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\cot 2\theta = (\cot^2\theta - 1) / 2 \cot\theta$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$\tan^2\theta = 1 + \cot^2\theta$$

$$\sin 2\theta = 2 \sin\theta \cos\theta$$

There are various formula for finding out sum and difference between two or more angles. These are called as difference and sum identities[6]. For two angles r and s these formula can be demonstrated as

$$\sin(r + s) = \sin(r)\cos(s) + \cos(r)\sin(s)$$

$$\tan(r+s) = \tan(r) + \tan(s)/(1 - \tan(r) \tan(s))$$

$$\sin(r - s) = \sin(r)\cos(s) - \cos(r)\sin(s)$$

$$\tan(r - s) = \tan(r) - \tan(s)/(1 + \tan(r) \tan(s))$$

$$\cos(r - s) = \cos(r)\cos(s) + \sin(r)\sin(s)$$

$$\cos(r + s) = \cos(r)\cos(s) - \sin(r)\sin(s)$$

For any three angles x, y, z and three sides X, Y, Z of a triangle there are Sine and Cosine laws which gives following relation:

1.2.1 Sine Laws:

$$x/\sin X = y/\sin Y = z/\sin Z$$

1.2.2 Cosine Laws:

$$z^2 = x^2 + y^2 - 2xy \cos Z$$

$$x^2 = y^2 + z^2 - 2yz \cos X$$

$$y^2 = x^2 + z^2 - 2zx \cos Y$$

There are Euler's formula in trigonometric which are used to find out exponential expressions. These are:

$$e^{ix} = \cos x + i \sin x$$

Where x = angle and i = imaginary number.

$$\tan x = (e^{ix} - e^{-ix}) / i(e^{ix} + e^{-ix})$$

$$\text{Sin}x = (e^{ix} - e^{-ix})/2i$$

$$\text{Cos}x = (e^{ix} + e^{-ix})/2i$$

1.3 Trigonometry Table:

Trigonometric table contains the most commonly used values of different functions[7].

TABLE 1: TABLE OF MOSTLY USED VALUES OF TRIGONOMETRIC FUNCTIONS AT DIFFERENT ANGLES

Angle	0	30	45	60	90
Cosec θ	∞	2	$\sqrt{2}$	$2/\sqrt{3}$	1
Cot θ	∞	$\sqrt{3}$	1	$1/\sqrt{3}$	0
Sec θ	1	$2/\sqrt{3}$	$\sqrt{2}$	2	∞
Cos θ	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
Sin θ	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
Tan θ	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞

Table 1 represents the mostly used values of Trigonometric Functions at different angles such as 0, 30, 45, 60, 90 and rest other values can be find out using these values with the help of trigonometric identities and relation between the functions.

1.4 Circular Representation using Unit Circle:

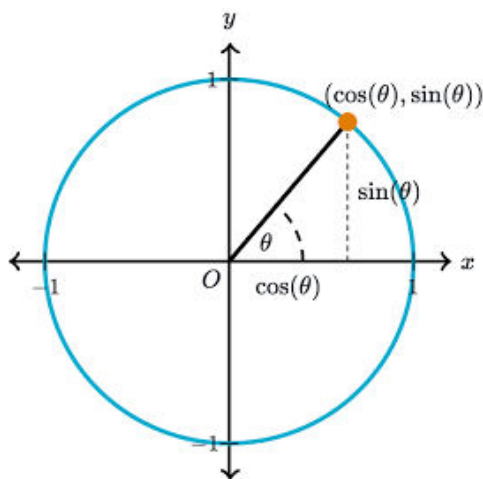


Figure 2: Circular representation of angle θ in triangle using Unit Circle[8].

In Unit Circle, Figure 2 displays a circular depiction of a triangle with sin and cos indicating its location coordinates. Because the circle's centre is at the origin, resulting in a radius of 1, it aids individuals in directly measuring the angles of tan, sin, and cos. The base of the triangle is p, and the perpendicular is q. The Hypotenuse will have the same length as the circle's radius, which is 1. Trigonometric ratios in a unit circle are as follows:

1.5 Angle of Elevation:

It's the angle formed when someone looks up horizontally from the ground. It is the angle that exists between the horizontal plane and the observer's eye. For example, if a woman stares at a tower from the ground, an angle is created that is inclined towards the tower from the woman's eye. Using trigonometric functions, this angle of elevation may be used to measure distances and heights of bridges, buildings, and other structures.

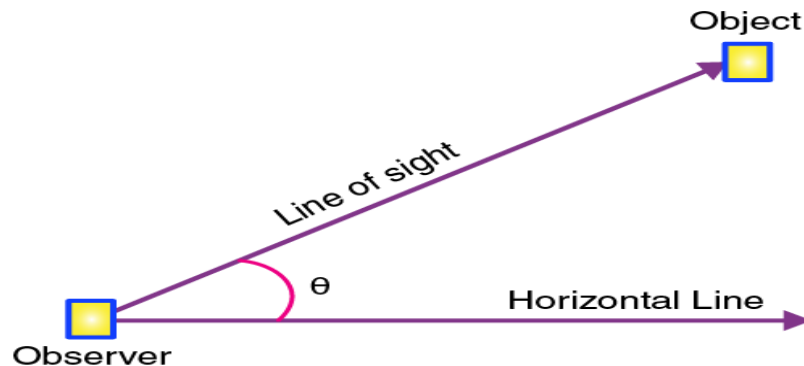


Figure 3: Angle of elevation from ground to an object higher than ground[9].

Figure 3 shows the Angle of elevation from ground to an object higher than ground with respect to the observer.

Formula to find out angle of elevation is:

$$\tan \theta = \text{Side Opposite to angle} / \text{Side Adjacent to angle}$$

1.6 Angle of Depression:

It's the angle formed when someone stares horizontally downwards from somewhere above ground level. It is the angle formed between the horizontal plane and the observer's eye when the observer is at a higher level than what he is looking at. For example, if a lady stares at a ball from terris, an angle is created that is slanted downward towards the ball from the woman's eye. Using trigonometric functions, this angle of depression is extremely helpful for calculating distances and heights of bridges, buildings, and other structures.

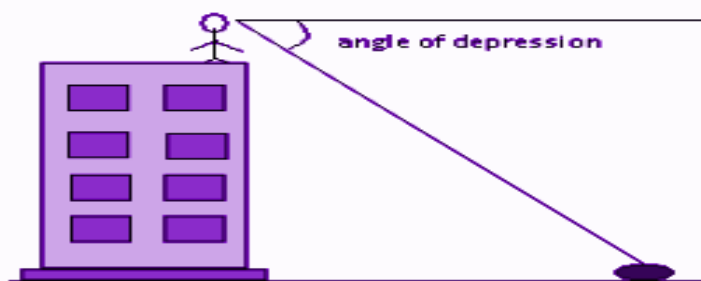


Figure 4: Woman staring from terris of building on the ball kept at ground resulting in formation of angle of depression[9].

Figure 4 shows a woman staring from terris of building on theball kept at ground resulting in formation of angle of depression.

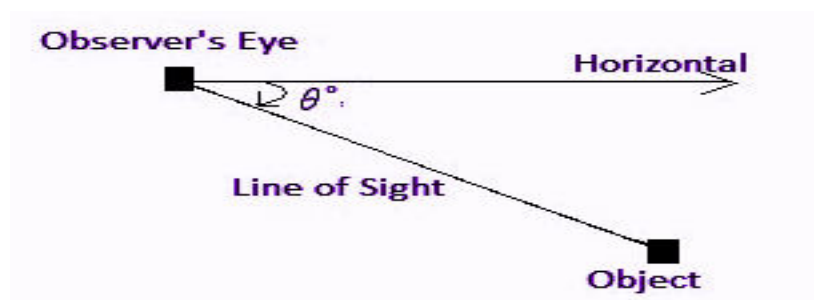


Figure 5: General representation of formation of angle of depression[9].

Figure 5 shows general representation of formation of angle of depression when someone looks at something kept at a lower level than the observer.

Formula to find out angle of depression cab be written as:

$$\tan \theta = \text{Side Opposite to angle} / \text{Side Adjacent to angle}$$

1.7 Graphical representation of Trigonometric Functions:

Trigonometric functions can be represented in form of waves.

1.7.1 Sine wave:

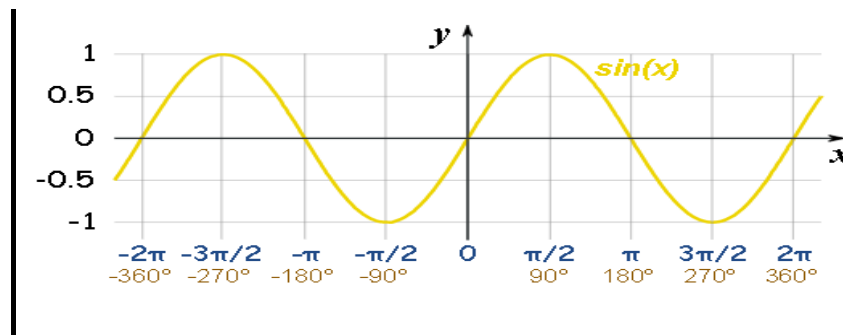


Figure 6: Graphical representation of Sine Function[10].

Figure 6 shows graphical representation of Sine function. In it X axis represent the values of function and Y axis represent angle. Sine function is continuous and passes through origin.

1.7.2 Cosine wave:

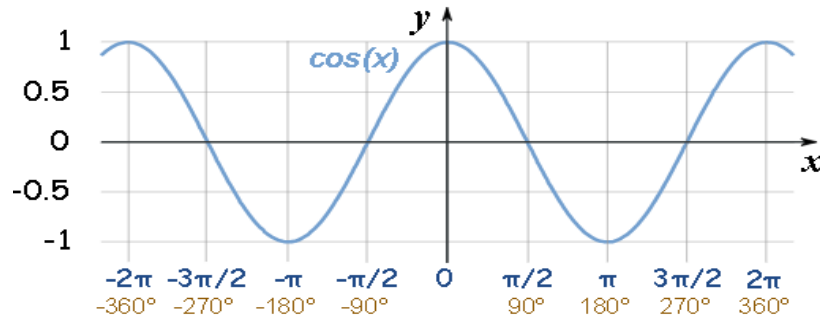


Figure 7: Graphical representation of Cosine Function[10].

The Cosine function is graphically shown in Figure 7. The X axis represents function values, whereas the Y axis represents angle. The sine function is a continuous function that does not go through the origin.

1.7.3 Tangent wave:

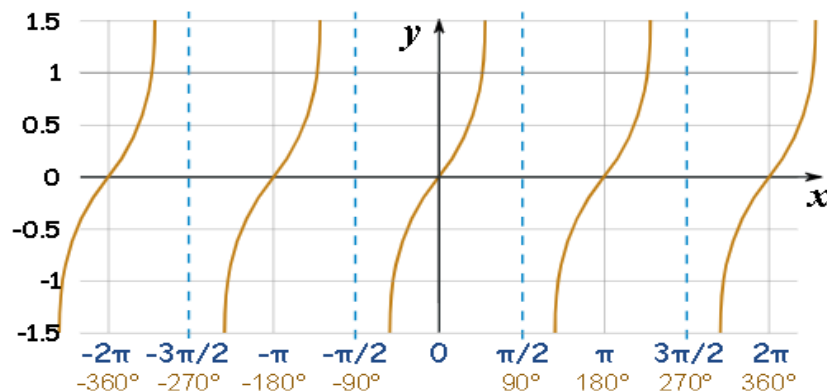


Figure 8: Graphical representation of Tangent Function[10]

The Tangent function is shown graphically in Figure 8. The X axis represents function values, whereas the Y axis represents angle. Tangent waves are not continuous waves that travel through the origin.

2. DISCUSSION

Mathematics is a discipline that studies numbers and the operations that may be done on them. It's a fascinating topic focused entirely on numbers, computations, and logic. It's a fascinating topic in which there's no need to memorize anything as in other courses; all that's required is attention and concentration when performing calculations. It offers a wide range of employment possibilities, which is why many students and parents choose Math as their primary subject in senior secondary school. Mathematics is built on a number of different ideas. Trigonometry is one of them, and it's a fascinating and entertaining subject with real-world applications. The

many features and functions of trigonometry are explained in this article. It contains all of the trigonometric formulas that may be used to real-world situations and solved. It teaches how to depict different trigonometric functions graphically. This article shows how mathematics may lead to a variety of employment possibilities, which can improve students' futures. In addition to all of this, the future scope of mathematics was addressed in this article.

3. CONCLUSION

Mathematics is a huge topic with a lot of different ideas to learn. The majority of mathematical progress has been practical. Mathematics is built on the foundation of numbers. Logic and numbers are the foundations of all mathematical theories. There are a number of initiatives that are entirely based on mathematical ideas. In the area of mathematics, there are many career possibilities such as speaker, professor, teacher, statistician, mathematician, operations research analyst, and many more. Sequences and Progressions, which include Arithmetic Progression, Harmonic Progression, and Geometric Progression, Trigonometry, Principle of Mathematical Induction, Binomial Equations, Differential Equation, Calculus, Differentiation and Integration, Probability, and more, are all concepts in mathematics. Geometry, which includes concepts such as the shapes, sizes, areas, and volumes of Triangles, Squares, Spheres, Cones, Cylinders, and many other geometrical figures, Complex Numbers, Sets, and many other concepts in mathematics, are used in real-life applications in areas such as building construction, bridge construction, and many other tasks.

Trigonometry is one such idea that is extensively utilized in real-world applications such as dam construction, bridge construction, building construction, road construction, and so on. In the construction of structures, the angle of elevation and depression are very significant. To determine the sides and angles of triangles with one 90-degree angle, trigonometry functions such as cosine, tangent, secant, sine, cosecant, and cotangent are employed. All of these formulas for finding sides and angles are covered in this article. In this article, many trigonometric identities and rules are explored. Mathematics is regarded as a topic that offers both students and instructors with many possibilities. As a result, mathematics has a bright future ahead of it, with numerous chances for students and instructors to demonstrate their abilities and acquire experience and knowledge.

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