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ERRORS MADE BY CHILDREN IN SUBTRACTION

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ABSTRACT

This paper discussed the errors made by children in subtraction with brining backward. The main objective of this study is to identify the error pattern among the primary school children. A paper pencil test was administered to 1824 children in Grades 4 and 5. Each paper consisted of 25 questions. An interview also was conducted with few children selected at randomly. The researcher marked the students' answers. He sorted out the wrong responses made by the children. Most of the errors identified were systematic errors. There were 10 types of errors identified in this study. Subtracting the smaller number on the right hand side from the larger number on the right hand side was the popular error pattern among the children.

KEYWORDS: *Subtraction, Errors, Systematic Errors*

INTRODUCTION

Mathematics is considered as an important subject in both the primary and secondary levels in Sri Lanka. A person who intends to follow the General Certificate of Education Advanced Level (GCE AL), should have passed Mathematics and the First Language in the General Certificate of Education Ordinary Level (GCE OL) examination. Thus Primary mathematics is the foundation for the secondary mathematics.

Sri Lankan Primary Mathematics syllabi consist of six main topics – Numbers, Mathematical Operations, Measurement, Money, Space and Shapes, and Data Handling. These concepts are introduced at Grade One and develop gradually up to Grade Five in the primary circle (Mukunthan, 2013). Algebra, number theory, geometry, and algebra are the four primary branches of mathematics. The usefulness of arithmetic for daily life is important, its instrumental role in other disciplines, the need for a basic knowledge in many professions and the important role of arithmetic in developing critical reasoning cannot be ignored (Mishra, 2020).The four mathematical operations –Addition, Subtraction, Multiplication and Division are included under the main topic Mathematical Operations (arithmetic) .The idea about subtraction is introducedin Grade 2. Subtraction using objects and figures (quantity not exceeding 19) and Subtracting a number not greater than 9 from another number not greater than 9 are in Grade 2 syllabus. Subtraction without bringing backward, of numbers not greater than 99 is introduced in Grade 3. Subtraction with bringing backward in one occasion (numbers not exceeding three digits) is included in the Grade 4 and Subtraction with bringing forward in two occasions (numbers not exceeding four digits) in Grade 5.

Addition and subtraction consist of additive relationships constituted of part-part-whole relations, and the ability to discern and handle this structure has been described as critical for children's development of powerful arithmetic strategies and skills (Baroody, 2016; Fritz, Ehlert, & Balzer, 2013; Resnick, 1983).

Subtraction involves more than applying an algorithm. It is not merely rote learning and mechanical process, but rather includes a process of acquiring the algorithmic procedure and interpreting that procedure, which has to be taken into account when and how the algorithm is applied and what it means. The importance of the subject understanding the conceptual basis of the algorithmic process of subtraction has been emphasized as essential in many studies. Several studies have highlighted that children make errors in subtraction.

LITERATURE REVIEW

Generally error can be defined as something a person has done which is reflected to be incorrect or wrong. Mathematical error can be defined as a mistake in a mathematical calculation. Children make several types of mathematical errors in the classrooms. According to Drew's (2005) error could be made due to many reasons. It could be the results of carelessness, misinterpretation of symbols or text, lack of relevant experience or knowledge of mathematical topic / learning objective / concepts, a lack of awareness or inability to check the answer given, or the result in misconception.

Mulhern and Greer (1989) noted that,

a) Errors are frequently 'surprising' usually because they may have remained undetected by teacher for some time.

b) Errors are often extremely 'persistent'. Typically, these are resistant to change themselves and correction of errors may involve fundamental reorganization of pupil's knowledge.

c) Frequently errors 'ignore meaning'. So that an answer which is obviously incorrect is rejected with no questioning.

Errors can occur in many ways, as described above. According to Brousseau et al(in Mulhern and Greer, 1989)errors may occur in four main ways in mathematics.

a) As a result of major misconceptions about fundamental aspects of mathematics

b) As a result of correct and faithful application of systematically flawed procedure

c) When the flawed rules and misconceptions the pupils possess are not recognized by the teacher.

d) Due to the use of highly original, non-formal methods of solving problems invented by pupils.

Mathematical errors are significant in practice because they can be used to enhance teaching and learning.

Fiori and Zuccherri (2005) found that certain error patterns depend on different ethnic-cultural situations, teaching methods and algorithm used, and to what extent.

Many researchers analyzed several types of mathematical errors. They have classified errors under few patterns. Cox (1975) classified the mathematical errors into three major categoriesviz (i) Careless error, (ii) Random error and (iii) Systematic error.

Cox (1975) defined the Careless error as: 'A student misses one or two problems out of five problems of a given type, random erroras: 'A student misses three or more problems out of five problems of a given type, but no pattern is apparent, and Systematic error as A student misses three or more problems out of five problems of a given type, using the same incorrect process as evidenced by the presence of a repeated pattern'.

Radatz (1979) classified the errors in terms of

(1) Language difficulties. Mathematics is like a "foreign language" for students who need to know and understand mathematical concepts, symbols, and vocabulary. Misunderstanding the semantics of mathematics language may cause students' errors at the beginning of problem solving;

(2) Difficulties in processing iconic and visual representation of mathematical knowledge;

(3) Deficiency in requisite skills, facts, and concepts; for example, students may forget or be unable to recall related information in solving problems;

(4) Incorrect associations or rigidity; that is, negative transfer caused by decoding and encoding information; and

(5) Application of irrelevant rules or strategies.

Department for Education and Employment (1999) of United Kingdom published a model for diagnosing children's error.

(i) Computational error / Careless mistake

(ii) Misconceptions

(iii) Wrong Operation

(iv) Over-generalization

(v) Under-generalization

(vi) Random response

Young and O'Shea (1981) noted that, many of the errors that occur in children's subtraction are due to the use of incorrect strategies rather than to the incorrect recall of number facts. A production system is presented for performing written subtraction, which is consistent with an earlier analysis of the nature of such a cognitive skill. Most of the incorrect strategies used by schoolchildren can be accounted for in a principled way by simple changes in the production system, such as the omission of individual rules or the inclusion of rules appropriate to other arithmetical tasks.

Children's difficulties in subtraction appear to be due, in part, to difficulties with their informal approach. Moreover, difficulties at the informal level may hinder the development of more advanced procedures such as reasoning out facts (e.g. 6 – 4is 2 because 2 is what has to be added to 4 to make 6) or recalling facts(Carpenter*et al* 1996).

Grossnickle & Snyder (1939) noted that, many students made careless mistakes in subtraction sums. Buswell & John (1926), Cox (1975), and Smith (1968) identified that, inversion errors are by far the most common type of all systematic errors. (Example: 95 – 38= 63, 73 – 29= 56).

The significant lack of understanding of basic mathematics is a circumstance that is accepted by the education community, while the continued use of algorithmic calculations remains a priority aim in the majority of countries worldwide. For example, subtraction tends to be considered, following rote learning, as a mechanical process that consists of applying algorithmic steps in the appropriate order.

Methodology and Results

The objective of this study is to identify the patterns of errors made by the students in subtraction with bringing backward.

There are five types of schools in Sri Lanka:National Schools, Type 1 AB, Type 1C, Type 2 and Type 3. The National Schools do not have primary sections. Sinhala, Tamil and English are the instructional media in these schools. All schools having that have primary sections were selected using Stratified Random Sampling Method for this Study. Accordingly, 25 schools were selected.

A question paper consisted of 25 subtraction sums with bringing backward. The Researcher designed the Question Paper. All the question consisted numbers less than 99. The second digit of the first number of each problem is greater the second digit of the second number in each problem. The question paper was administered to 1824 students in Grade 4 and 5selected from these 25 schools. The duration to answer the questions was One hour and twenty minutes. All the sums (1824 X 25 =45600)were marked by the researcher. Then the researcher sort out the sums with the errors. Then he identified the pattern of errors from those sums.

Only 138 students answered all the questions correctly. The sample answered 26137 sums out of the 45600 sums correctly. Therefore 19,463 (42.68%)answers were wrong. There were 23 students who did not answer any questions. Therefore the balance 18888 answers were analyzed by the researcher. 200 students were randomly selected and interviewed and the reason for their answers were inquired by the researcher.

According to Cox *(1975) children's errors were sort out as random errors, careless errors and systematic errors. The type of errors and the percentages are given in below Table*

TABLE 1 ERROR TYPE AND STUDENTS NUMBERS

Error Type	Number	Percentage
Careless Error	434	2.30
Random Error	368	1.95
Systematic Error	18086	95.75
Total	18888	100.00

There systematic errors (18,806) were further grouped.

Analyzing the Systematic Errors

According to Nanayakkara (1992) Systematic Errors could have arisen for many reasons. Data collected during interviews enabled to find the most important reason. Observation of the children's answers and the interview with them enabled the researcher to identify the following eight error types.

The question format is given bellow:

ab

-cd

…..

Where 'a' is greater than 'c'.

The identified students' error patterns are as follows:

(i) Error type I (recalling error)

Recalls subtraction basic facts incorrectly.

Example:

76	83	60	91	52
-59	-47	-36	-28	-13
…..	…….	…..	……	……
22	45	37	77	47

(ii) Error Type II(direction of operation)

Starts the subtraction from the left side.

Example

76	83	60	91	52
-59	-47	-36	-28	-13

…..	…….	…..	……	……
2	4	3	7	4

(iii) Error Type III(Zero related error)

Subtract incorrectly when zero is included as one of the numbers.

Example:

76	80	60	91	50
-50	-47	-36	-20	-13
…..	…….	…..	……	……
20	40	30	70	40

(iv) Error Type IV

Bringing one from the 10th place and subtracted correctly but when subtracting in the 10th placeforgot the number that was brought backward.

Example

76	83	60	91	52
-59	-47	-36	-28	-13
…..	…….	…..	……	……
27	46	34	73	49

(v) Error Type V

Adds both numbers.

Example

76	83	60	91	52
-59	-47	-36	-28	-13
…..	…….	…..	……	……
135	13 0	96	119	65

Sign problem.

(vi) Type VI (b – d = d – c)

Example

76	83	60	91	52
-59	-47	-36	-28	-13
…..	…….	…..	……	……
2 3	44	36	77	41

Subtracts the small number of the right hand side from the big number of the right hand side.

(vii). Type VII error b – d = b

Example

76	83	60	91	52
-59	-47	-36	-28	-13
…..	…….	…..	……	……
26	43	30	71	42

Writes the small number.

(viii) Error Type VIII (b – d = d)

Example

76	83	60	91	52
-59	-47	-36	-28	-13
…..	…….	…..	……	……
29	47	36	78	43

Writes the larger number.

(ix) Error Type IX (b – d = 0)

If they cannot subtract they just write 0

Example

76	83	60	91	52
-59	-47	-36	-28	-13
…..	…….	…..	……	……
20	40	30	70	40

(x) Error Type X (b – d = ..)

If they cannot subtract they just leave it blank.

Example

76	83	60	91	52
-59	-47	-36	-28	-13
…..	…….	…..	……	……
2	4	3	7	4

Writes the larger number

TABLE 2

Error type in systematic errors	**numbers**	**percentages**
Error Type I	1002	5.54
Error Type II	2171	12.00
Error Type III	1418	7.84

Error Type IV	1820	10.06
Error Type V	1112	6.15
Error Type VI	8471	46.84
Error Type VII	1735	9.59
Error Type VIII	90	0.50
Error Type IX	201	1.11
Error Type X	66	0.36
Total	**18086**	**100.00**

(E) Discussion And Conclusion

As already noted, Cox (1975) defined that, systematic error as a student misses three or more problems out of five problems of a given type, using the same incorrect process as evidenced by the presence of a repeated pattern. According to Cox's error classification, more errors (95.7%) found in this study are systematic errors.

Error types VI – X are related to borrow the number from the 10th place.

Errors induced by a borrow: (a) Misreporting of higher-order minuend values. (b) Forgetting the tens-column digit of the subtrahend, after executing a borrow. (c) Errors within the borrow operation itself, depending on location of the borrow, forgetting the tens-column value while executing the unit-columns addition of 10 and fact retrieval, and (d) forgetting to decrement the tens-column value in completing the borrow (Kase et al 2006) .

Further, systematic errors analyzed, showed that 46.84% of the errors were Error Type VI. That is children subtract the smaller number from the larger number in the right hand side. When the children do the addition or multiplication sums they do not need to consider the order of the numbers (example 2+3 = 3+2, 4x 5 = 5 x 4). But in the operation subtraction and division this should be consider. Children may confuse this order of the operation.

12% of the errors in the sample arethe Error Type II (direction of operation). When the children read or write in language they starts from left to right. But in the mathematical operations addition, subtraction and multiplication start from the right hand side. This may be the reason for children's error on direction of operation in the subtraction.

10.06% of the errors in the sample were related to sign. Children add both numbers. Because they did not consider the sign of the operation. Children learned addition first then they learned subtraction. When they do addition, they only have to add. Therefore, they did not seriously consider the sign but simply add the numbers. When they start subtracting the numbers,they, again, add the numbers.

The wrong answers for the Error Type II (Directional Error)and Error Type IX (writing zero)are the same. But the method of getting the answers are different. This error pattern was identified through the interview.

Finally it can be concluded that most of the children made systematic errors. If the bottom number is larger than the top number many children subtract the numbers from the bottom to top.

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