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A GRAPH IN THE FORM OF A TRIANGLE WITH ATTACHED OUTGOING EDGES AT EACH VERTEX

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ABSTRACT

We study one incoming, two outgoing and triangle graphs for the equation of linear KdV. Using the theory of potentials, we reduce the problem to systems of linear integral equations and show that they are uniquely solvable under conditions of the uniqueness theorem.

KEYWORDS: *Third Order PDE, Boundary Value Problem, Method Of Energy Integrals, Method Of Potentials, Initial Condition, Boundary Condition, Integral Equation.*

INTRODUCTION

In this paper, we address the linearized KdV equation on a star graph Γ with one bounded bond and two semi-infinite bonds connected at one point, called the vertex.

The bonds are denoted by $B_j,\ j=\overline{1,6}$ the coordinate x_1 on B_1 is defined from $-L_1$ to 0, and coordinates x_2 and x_3 and x_4 from links B_2 and B_3 and B_4 from 0 to L,

and coordinates x_5 and x_6 on the bonds B_5 and B_6 are defined from 0 to such that on each bond the vertex corresponds to 0. On each bond we consider the linear equation:

$$\left(\frac{\partial}{\partial t}-\frac{\partial^3}{\partial x_j^3}\right)u_j(x_j,t)=f_j(x,t),\quad t>0, x_j\in B_j,\ j=\overline{1,6}. \qquad (1)$$

Below, we will also use the notation x instead of $x_j,\ j=\overline{1,6}$. We treat a boundary value problem and using the method of potentials, reduce it to a system of integral equations. The solvability of the obtained system of integral equations is proven.

1. Formulation of the problems

To solve the linear KdV equation on an interval, one needs to impose three boundary conditions (BC): two on the left end of thex-interval and one on the right end, (see, e.g.,[5-6] and references therein). For the above star graph, we need to impose 5 BCs at thevertex point, which should provide also connection between the bonds and 2 BCs at the leftside of B_1. In detail, we require

$$\begin{cases} u_1(0,t)=u_2(0,t)=u_3(0,t),\ u_{1x}(0,t)=\dfrac{1}{b_2}u_{2x}(0,t)=\dfrac{1}{b_3}u_{3x}(0,t), \\ u_{1xx}(0,t)=u_{2xx}(0,t)+u_{3xx}(0,t),\ u_1(-L_1;t)=\phi_1(t), \end{cases} \quad (2)$$

$$\begin{cases} u_2(L_2,t)=u_4(0,t)=u_5(0,t),\ u_{2x}(L_2,t)=\dfrac{1}{b_4}u_{4x}(0,t)=\dfrac{1}{b_5}u_{5x}(0,t), \\ u_{2xx}(L_2,t)=u_{4xx}(0,t)+u_{5xx}(0,t), \end{cases} \quad (3)$$

$$\begin{cases} u_3(L_3,t)=u_4(L_4,t)=u_6(0,t),\ u_{3x}(L_3,t)=\dfrac{1}{b_4}u_{4x}(L_4,t)+\dfrac{1}{b_6}u_{6x}(0,t), \\ u_{3xx}(L_3,t)=u_{4xx}(L_4,t)+u_{6xx}(0,t), \end{cases} \quad (4)$$

for $0<t<T$, $T=const$.Furthermore, we assume that the functions $f_j(x,t),\ j=\overline{1,6}$ are smooth enough hand bounded. The initial conditions are given by:

$$u_j(x,0)=u_{0,j}(x),\quad x\in\overline{B_j},\quad j=\overline{1,6}. \quad (5)$$

It should be noted that the above vertex conditions are not the only possible ones. The main motivation for our choice is caused by the fact that they guarantee uniqueness of the solution and, if the solutions decay (to zero) at infinity, the norm (energy) conservation.

2.Existence and uniqueness of solutions

Lemma 1. Let $b_2^2+b_3^2\le 1, b_4^2+b_5^2\le 1,\ b_4^2+b_6^2\le 1$ Then the (1)-(5) has at most one solution.

Prof of Lemma 1. Using the equation (1) one can easily get:

$$\frac{\partial}{\partial t}\int_a^b u_j^2(x,t)dx=\left(2u_ju_{jxx}-u_{jx}^2\right)\Big|_{x=a}^{x=b}+2\int_a^b f_j(x,t)u_j(x,t)dx$$

for appropriate values of constants a and b on each bond. The, the above equalities and vertex conditions (2)-(5) yield:

$$\frac{\partial}{\partial t}\left(e^{-\varepsilon t}\|u\|^2\right) \le e^{-\varepsilon t}\left(\frac{1}{\varepsilon^2}\|f\|^2 + \phi_1^2(t)\right),$$

$$\|u\|^2 \le \|u_0\|^2 + \int_0^t e^{-\varepsilon(t-\tau)}\left(\frac{1}{\varepsilon^2}\|f(\cdot,\tau)\|^2 + \phi_1^2(\tau)\right)d\tau \ , \qquad (6)$$

Where

$$(u,v) = \int_{-L_1}^{0} u_1v_1dx_1 + \int_0^{L_2} u_2v_2dx_2 + \int_0^{L_3} u_3v_3dx_3 + \int_0^{L_4} u_4v_4dx_4 + \int_0^{+\infty} u_5v_5dx_5 + \int_0^{+\infty} u_6v_6dx_6 \ ,$$

$\|u\| = \sqrt{(u,u)}$ are L_2 scalar product and norm defined on graph, ε -is an arbitrary positive number.

Uniqueness of the solution follows from (6).

Lemma 2. a) Let $\omega \in [a;b]$. Then $u(x,t)$ satisfies $u_t - u_{xxx} = 0$ for $t > 0$ and:

$$\lim_{(x,y)\to(x_0,0)} u(x,t) = \begin{cases} \pi\omega(x_0), & if \ x_0 \in (a,b); \\ 0, & if \ x_0 \notin (a,b). \end{cases}$$

b) Let $f \in L^2((a,b)\times(0,T))$. Then, $v(x,t)$ satisfies $u_t - u_{xxx} = \pi f(x,t)$ in $(a,b)\times(0,T]$, $T > 0$ and initial condition $u(x,0) = u_0(x), \ x \in (a,b)$.

c) If $\varphi_k \in H^1(0,T)$, then

$$\lim_{x\to a+0} \omega^{(1)}(x,t) = \frac{2\pi}{3}\varphi(y), \ \lim_{x\to a-0} \omega^{(1)}(x,t) = -\frac{\pi}{3}\varphi(y), \ \lim_{x\to a+0} \omega^{(2)}(x,t) = 0$$

Now, we are ready to construct exact solutions for the considered problems. We assume that initial data and source terms in each bond are sufficiently smooth and bounded functions.[16]

Main results

Theorem 1. Let $\det A \neq 0, \ \varphi_k \in C^1[0,T]$

Then the problem (1)-(5) has a unique solution in. $u_{0,j}(x) \in C^3[0,T]$

$(C^1([0,\ T],\ C^3(\Gamma)))$

Proof of Theorem 1.To prove the theorem, we use the following functions are called fundamental solutions of the equation $u_t - u_{xxx} = 0$. (see [1, 3, 5, 12, 16]):

$$U(x,t;\xi,\eta)=\begin{cases}\dfrac{1}{(t-\eta)^{\frac{1}{3}}}f\left(\dfrac{x-\xi}{(t-\eta)^{\frac{1}{3}}}\right), & t>\eta,\\ 0 & t\le\eta\end{cases}$$

$$V(x,t;\xi,\eta)=\begin{cases}\dfrac{1}{(t-\eta)^{\frac{1}{3}}}\varphi\left(\dfrac{x-\xi}{(t-\eta)^{\frac{1}{3}}}\right), & t>\eta,\\ 0 & t\le\eta\end{cases}$$

where $f(x)=\frac{\pi}{\sqrt[3]{3}}Ai\left(-\frac{x}{\sqrt[3]{3}}\right)$, $\varphi(x)=\frac{\pi}{\sqrt[3]{3}}Bi\left(-\frac{x}{\sqrt[3]{3}}\right)$ for $x\ge 0$, $\varphi(x)=0$ for $x<0$ and $Ai(x)$ and $Bi(x)$ are the Airy functions. The functions $f(x)$ and $\varphi(x)$ are integrable and $\int\limits_{-\infty}^{0} f(x)dx=\frac{\pi}{3}$, $\int\limits_{0}^{+\infty} f(x)dx=\frac{2\pi}{3}$, $\int\limits_{0}^{+\infty}\varphi(x)dx=0.$

We summarize some properties of potentials for (1) from [3,5]. Forgiven ω, f and φ let:

$$u(x,t)=\int\limits_a^b U(x,t;\xi,0)\omega(\xi)d\xi,\ v(x,t)=\int\limits_0^t\int\limits_a^b U(x,t;\xi,\tau)f(\xi,\tau)d\xi d\tau,$$

$$\omega^{(1)}(x,t)=\int\limits_0^t U_{x\xi}(x,\eta;a,t)\varphi(\eta)d\eta,\ \omega^{(2)}(x,t)=\int\limits_0^t V_{x\xi}(x,\eta;a,t)\varphi(\eta)d\eta.$$

Below, we also use fractional integrals [8]

$$J_{(0,t)}^{\alpha}f(t):=\frac{1}{\Gamma(\alpha)}\int\limits_0^t(t-\tau)^{\alpha-1}f(\tau)d\tau,\ 0<\alpha<1$$

and the inverse of this operator, i.e. the Riemann-Liouville fractional derivatives [8, 9]defined by: $D_{(0,t)}^{\alpha}f(t):=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int\limits_0^t(t-\tau)^{-\alpha}f(\tau)d\tau,\ 0<\alpha<1.$

We look for solution in the form:

$$u_1(x,t)=\int\limits_0^t U(x,t;0,\eta)\varphi_1(\eta)d\eta+\int\limits_0^t U(x,t;-L_1,\eta)\alpha_1(\eta)d\eta+F_1(x,t)$$

$$u_k(x,t)=\int_0^t U(x,t;0,\eta)\varphi_k(\eta)d\eta+\int_0^t V(x,t;0,\eta)\alpha_k(\eta)d\eta+$$
$$+\int_0^t U(x,t;L_k,\eta)\beta_k(\eta)d\eta+F_k(x,t),\ k=2,3,4.$$

$$u_5(x,t)=\int_0^t U(x,t;0,\eta)\varphi_5(\eta)d\eta+\int_0^t V(x,t;0,\eta)\psi_5(\eta)d\eta+F_5(x,t),$$
$$u_6(x,t)=\int_0^t U(x,t;0,\eta)\varphi_6(\eta)d\eta+\int_0^t V(x,t;0,\eta)\psi_6(\eta)d\eta+F_6(x,t),$$

Where

$$F_k(x,t)=\frac{1}{\pi}\int_0^t\int_{B_k} U(x,t;\xi,\eta)f_k(\xi,\eta)d\xi d\eta,\ k=\overline{1,6} \qquad (7)$$

Satisfying the conditions (2) we have:

$$a)\ u_1(0,t)=u_2(0,t)=u_3(0,t),$$

$$f(0)\varphi_1(t)-f(0)\varphi_2(t)-\varphi(0)\alpha_2(t)+$$
$$+\frac{1}{\Gamma\left(\frac{1}{3}\right)}D_{(0,t)}^{2/3}\int_0^t\frac{1}{(t-\eta)^{1/3}}f\left(\frac{L_1}{(t-\eta)^{1/3}}\right)\alpha_1(\eta)d\eta-$$
$$-\frac{1}{\Gamma\left(\frac{1}{3}\right)}D_{(0,t)}^{2/3}\int_0^t\frac{1}{(t-\eta)^{1/3}}f\left(\frac{-L_2}{(t-\eta)^{1/3}}\right)\beta_2(\eta)d\eta= \qquad (8)$$
$$=\frac{1}{\Gamma\left(\frac{1}{3}\right)}D_{(0,t)}^{2/3}\left[F_2(0,t)-F_1(0,t)\right]$$

(8*) can be derived from (8)

$$f(0)\varphi_1(t)-f(0)\varphi_2(t)-\varphi(0)\alpha_2(t)+$$
$$+\frac{1}{\Gamma\left(\frac{1}{3}\right)}\int_0^t K_1\alpha_1(\eta)d\eta-\frac{1}{\Gamma\left(\frac{1}{3}\right)}\int_0^t K_2\beta_2(\eta)d\eta= \qquad (8^*)$$
$$=\frac{1}{\Gamma\left(\frac{1}{3}\right)}D_{(0,t)}^{2/3}\left[F_2(0,t)-F_1(0,t)\right]$$

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$$f(0)\varphi_1(t) - f(0)\varphi_3(t) - \varphi(0)\alpha_3(t) +$$
$$+\frac{1}{\Gamma\left(\frac{1}{3}\right)} D^{2/3}_{(0,t)} \int_0^t \frac{1}{(t-\eta)^{1/3}} f\left(\frac{L_1}{(t-\eta)^{1/3}}\right)\alpha_1(\eta)d\eta - \tag{9}$$
$$-\frac{1}{\Gamma\left(\frac{1}{3}\right)} D^{2/3}_{(0,t)} \int_0^t \frac{1}{(t-\eta)^{1/3}} f\left(\frac{-L_3}{(t-\eta)^{1/3}}\right)\beta_3(\eta)d\eta =$$
$$= \frac{1}{\Gamma\left(\frac{1}{3}\right)} D^{2/3}_{(0,t)} \left[F_3(0,t) - F_1(0,t)\right]$$

(9*) can be derived from (9)

$$f(0)\varphi_1(t) - f(0)\varphi_3(t) - \varphi(0)\alpha_3(t) +$$
$$+\frac{1}{\Gamma\left(\frac{1}{3}\right)} \int_0^t K_3\alpha_1(\eta)d\eta - \frac{1}{\Gamma\left(\frac{1}{3}\right)} \int_0^t K_4\beta_3(\eta)d\eta = \tag{9*}$$
$$= \frac{1}{\Gamma\left(\frac{1}{3}\right)} D^{2/3}_{(0,t)} \left[F_3(0,t) - F_1(0,t)\right]$$

$$b) u_{1x}(0,t) = \frac{1}{b_2} u_{2x}(0,t) = \frac{1}{b_3} u_{3x}(0,t),$$

$$f'(0)\varphi_1(t) - \frac{1}{b_2} f'(0)\varphi_2(t) - \frac{1}{b_2}\varphi'(0)\alpha_2(t) +$$
$$+\frac{1}{\Gamma\left(\frac{2}{3}\right)} D^{1/3}_{(0,t)} \int_0^t \frac{1}{(t-\eta)^{2/3}} f'\left(\frac{L_1}{(t-\eta)^{1/3}}\right)\alpha_1(\eta)d\eta - \tag{10}$$
$$-\frac{1}{b_2\Gamma\left(\frac{2}{3}\right)} D^{1/3}_{(0,t)} \int_0^t \frac{1}{(t-\eta)^{2/3}} f'\left(\frac{-L_2}{(t-\eta)^{1/3}}\right)\beta_2(\eta)d\eta =$$
$$= \frac{1}{\Gamma\left(\frac{2}{3}\right)} D^{1/3}_{(0,t)} \left[\frac{1}{b_2} F_{2x}(0,t) - F_{1x}(0,t)\right]$$

(10*) can be derived from (10)

$$f'(0)\varphi_1(t)-\frac{1}{b_2}f'(0)\varphi_2(t)-\frac{1}{b_2}\varphi'(0)\alpha_2(t)+$$

$$+\frac{1}{\Gamma\left(\frac{2}{3}\right)}\int_0^t K_5\alpha_1(\eta)d\eta-\frac{1}{b_2\Gamma\left(\frac{2}{3}\right)}\int_0^t K_6\beta_2(\eta)d\eta= \qquad (10^*)$$

$$=\frac{1}{\Gamma\left(\frac{2}{3}\right)}D_{(0,t)}^{1/3}\left[\frac{1}{b_2}F_{2x}(0,t)-F_{1x}(0,t)\right]$$

$$f'(0)\varphi_1(t)-\frac{1}{b_3}f'(0)\varphi_3(t)-\frac{1}{b_3}\varphi'(0)\alpha_3(t)+$$

$$+\frac{1}{\Gamma\left(\frac{2}{3}\right)}D_{(0,t)}^{1/3}\int_0^t\frac{1}{(t-\eta)^{2/3}}f'\left(\frac{L_1}{(t-\eta)^{1/3}}\right)\alpha_1(\eta)d\eta- \qquad (11)$$

$$-\frac{1}{b_3\Gamma\left(\frac{2}{3}\right)}D_{(0,t)}^{1/3}\int_0^t\frac{1}{(t-\eta)^{2/3}}f'\left(\frac{-L_3}{(t-\eta)^{1/3}}\right)\beta_3(\eta)d\eta=$$

$$=\frac{1}{\Gamma\left(\frac{2}{3}\right)}D_{(0,t)}^{1/3}\left[\frac{1}{b_3}F_{3x}(0,t)-F_{1x}(0,t)\right]$$

(11*) can be derived from (11)

$$f'(0)\varphi_1(t)-\frac{1}{b_3}f'(0)\varphi_3(t)-\frac{1}{b_3}\varphi'(0)\alpha_3(t)+$$

$$+\frac{1}{\Gamma\left(\frac{2}{3}\right)}\int_0^t K_7\alpha_1(\eta)d\eta-\frac{1}{b_3\Gamma\left(\frac{2}{3}\right)}\int_0^t K_8\beta_3(\eta)d\eta= \qquad (11^*)$$

$$=\frac{1}{\Gamma\left(\frac{2}{3}\right)}D_{(0,t)}^{1/3}\left[\frac{1}{b_3}F_{3x}(0,t)-F_{1x}(0,t)\right]$$

c) $u_{1xx}(0,t)=u_{2xx}(0,t)+u_{3xx}(0,t),$

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$$-\frac{\pi}{3}\varphi_1(t)-\frac{2\pi}{3}\varphi_2(t)-\frac{2\pi}{3}\varphi_3(t)-$$
$$-\int_0^t\frac{1}{t-\eta}f''\left(-\frac{L_2}{(t-\eta)^{\frac{1}{3}}}\right)\beta_2(\eta)d\eta-\qquad(12)$$
$$-\int_0^t\frac{1}{t-\eta}f''\left(-\frac{L_3}{(t-\eta)^{\frac{1}{3}}}\right)\beta_3(\eta)d\eta+$$
$$+\int_0^t\frac{1}{t-\eta}f''\left(\frac{L_1}{(t-\eta)^{\frac{1}{3}}}\alpha_1(\eta)\right)d\eta$$
$$=F_{3xx}(0,t)+F_{2xx}(0,t)-F_{1xx}(0,t),$$

(12*) can be derived from (12)

$$-\frac{\pi}{3}\varphi_1(t)-\frac{2\pi}{3}\varphi_2(t)-\frac{2\pi}{3}\varphi_3(t)-$$
$$-\int_0^t K_9\beta_2(\eta)d\eta-\int_0^t K_{10}\beta_3(\eta)d\eta+\int_0^t K_{11}\beta_3(\eta)d\eta=\qquad(12^*)$$
$$=F_{3xx}(0,t)+F_{2xx}(0,t)-F_{1xx}(0,t),$$

d) $u_1(-L_1;t)=\phi_1(t)$

$$f(0)\alpha_1(t)-\frac{1}{\Gamma\left(\frac{1}{3}\right)}D_{(0,t)}^{2/3}\int_0^t\frac{1}{(t-\eta)^{1/3}}f\left(\frac{-L_1}{(t-\eta)^{1/3}}\right)\varphi_1(\eta)d\eta=$$
$$=\frac{1}{\Gamma\left(\frac{1}{3}\right)}D_{(0,t)}^{2/3}\left[\phi_1(t)-F_1(-L_1,t)\right]\qquad(13)$$

(13*) can be derived from (13)

$$f(0)\alpha_1(t)-\frac{1}{\Gamma\left(\frac{1}{3}\right)}\int_0^t K_{12}\varphi_1(\eta)d\eta=$$
$$=\frac{1}{\Gamma\left(\frac{1}{3}\right)}D_{(0,t)}^{2/3}\left[\phi_1(t)-F_1(-L_1,t)\right]\qquad(13^*)$$

Satisfying the conditions (3) we have:

$a)\ u_2(L_2,t)=u_4(0,t)=u_5(0,t),$

$$f(0)\beta_2(t) - f(0)\varphi_4(t) - \varphi(0)\alpha_4(t) +$$

$$+\frac{1}{\Gamma\left(\frac{1}{3}\right)} D^{2/3}_{(0,t)} \int_0^t \frac{1}{(t-\eta)^{1/3}} f\left(\frac{L_2}{(t-\eta)^{1/3}}\right)\varphi_2(\eta)d\eta +$$

$$+\frac{1}{\Gamma\left(\frac{1}{3}\right)} D^{2/3}_{(0,t)} \int_0^t \frac{1}{(t-\eta)^{1/3}} \varphi\left(\frac{L_2}{(t-\eta)^{1/3}}\right)\alpha_2(\eta)d\eta -$$

$$-\frac{1}{\Gamma\left(\frac{1}{3}\right)} D^{2/3}_{(0,t)} \int_0^t \frac{1}{(t-\eta)^{1/3}} f\left(\frac{-L_4}{(t-\eta)^{1/3}}\right)\beta_4(\eta)d\eta =$$

$$= \frac{1}{\Gamma\left(\frac{1}{3}\right)} D^{2/3}_{(0,t)} \left[F_4(0,t) - F_2(L_2,t)\right] \qquad (14)$$

(14*) can be derived from (14)

$$f(0)\beta_2(t) - f(0)\varphi_4(t) - \varphi(0)\alpha_4(t)$$

$$+\frac{1}{\Gamma\left(\frac{1}{3}\right)}\left[\int_0^t K_{13}\varphi_2(\eta)d\eta + \int_0^t K_{14}\alpha_2(\eta)d\eta + \int_0^t K_{15}\beta_4(\eta)d\eta\right] = \qquad (14^*)$$

$$= \frac{1}{\Gamma\left(\frac{1}{3}\right)} D^{2/3}_{(0,t)} \left[F_4(0,t) - F_2(L_2,t)\right]$$

$$f(0)\beta_2(t) - f(0)\varphi_5(t) - \varphi(0)\psi_5(t) +$$

$$+\frac{1}{\Gamma\left(\frac{1}{3}\right)} D^{2/3}_{(0,t)} \int_0^t \frac{1}{(t-\eta)^{1/3}} \left[f\left(\frac{L_2}{(t-\eta)^{1/3}}\right)\varphi_2(\eta) - \varphi\left(\frac{L_2}{(t-\eta)^{1/3}}\right)\alpha_2(\eta)\right]d\eta \qquad (15)$$

$$= \frac{1}{\Gamma\left(\frac{1}{3}\right)} D^{2/3}_{(0,t)} \left[F_5(0,t) - F_2(L_2,t)\right]$$

(15*) can be derived from (15)

$$f(0)\beta_2(t)-f(0)\varphi_5(t)-\varphi(0)\psi_5(t)+$$

$$+\frac{1}{\Gamma\left(\frac{1}{3}\right)}\left[\int_0^t K_{16}\varphi_2(\eta)d\eta+\int_0^t K_{17}\alpha_2(\eta)d\eta\right] \quad (15^*)$$

$$=\frac{1}{\Gamma\left(\frac{1}{3}\right)}D_{(0,t)}^{2/3}\left[F_5(0,t)-F_2(L_2,t)\right]$$

$$b)\ u_{2x}(L_2,t)=\frac{1}{b_4}u_{4x}(0,t)=\frac{1}{b_5}u_{5x}(0,t),$$

$$f'(0)\beta_2(t)-\frac{1}{b_4}f'(0)\varphi_4(t)-\frac{1}{b_4}\varphi'(0)\alpha_4(t)+$$

$$+\frac{1}{b_4\Gamma\left(\frac{2}{3}\right)}D_{(0,t)}^{1/3}\int_0^t\frac{1}{(t-\eta)^{2/3}}f'\left(\frac{L_2}{(t-\eta)^{1/3}}\right)\varphi_2(\eta)d\eta+$$

$$+\frac{1}{b_4\Gamma\left(\frac{2}{3}\right)}D_{(0,t)}^{1/3}\int_0^t\frac{1}{(t-\eta)^{2/3}}\varphi'\left(\frac{L_2}{(t-\eta)^{1/3}}\right)\alpha_2(\eta)d\eta-$$

$$-\frac{1}{b_4\Gamma\left(\frac{2}{3}\right)}D_{(0,t)}^{1/3}\int_0^t\frac{1}{(t-\eta)^{2/3}}f'\left(\frac{-L_4}{(t-\eta)^{1/3}}\right)\beta_4(\eta)d\eta$$

$$=\frac{1}{\Gamma\left(\frac{2}{3}\right)}D_{(0,t)}^{1/3}\left[\frac{1}{b_4}F_{4x}(0,t)-F_{2x}(L_2,t)\right] \quad (16)$$

(16*) can be derived from (16)

$$f'(0)\beta_2(t)-\frac{1}{b_4}f'(0)\varphi_4(t)-\frac{1}{b_4}\varphi'(0)\alpha_4(t)+$$

$$+\frac{1}{b_4\Gamma\left(\frac{2}{3}\right)}\left[\int_0^t K_{18}\varphi_2(\eta)d\eta+\int_0^t K_{19}\alpha_2(\eta)d\eta-\int_0^t K_{20}\beta_4(\eta)d\eta\right]= \quad (16^*)$$

$$=\frac{1}{\Gamma\left(\frac{2}{3}\right)}D_{(0,t)}^{1/3}\left[\frac{1}{b_4}F_{4x}(0,t)-F_{2x}(L_2,t)\right]$$

$$f'(0)\beta_2(t)-\frac{1}{b_5}f'(0)\varphi_5(t)-\frac{1}{b_5}\varphi'(0)\psi_5(t)+$$

$$+\frac{1}{b_5\Gamma\left(\frac{2}{3}\right)}D^{1/3}_{(0,t)}\int_0^t\frac{1}{(t-\eta)^{2/3}}f'\left(\frac{L_2}{(t-\eta)^{1/3}}\right)\varphi_2(\eta)d\eta= \qquad (17)$$

$$+\frac{1}{b_5\Gamma\left(\frac{2}{3}\right)}D^{1/3}_{(0,t)}\int_0^t\frac{1}{(t-\eta)^{2/3}}\varphi'\left(\frac{L_2}{(t-\eta)^{1/3}}\right)\alpha_2(\eta)d\eta$$

$$=\frac{1}{\Gamma\left(\frac{2}{3}\right)}D^{1/3}_{(0,t)}\left[\frac{1}{b_5}F_{5x}(0,t)-F_{2x}(L_2,t)\right]$$

(17*) can be derived from (17)

$$f'(0)\beta_2(t)-\frac{1}{b_5}f'(0)\varphi_5(t)-\frac{1}{b_5}\varphi'(0)\psi_5(t)+$$

$$+\frac{1}{b_5\Gamma\left(\frac{2}{3}\right)}\left[\int_0^t K_{21}\varphi_2(\eta)d\eta+\int_0^t K_{22}\alpha_2(\eta)d\eta\right]= \qquad (17^*)$$

$$=\frac{1}{\Gamma\left(\frac{2}{3}\right)}D^{1/3}_{(0,t)}\left[\frac{1}{b_5}F_{5x}(0,t)-F_{2x}(L_2,t)\right]$$

$$c)\ u_{2xx}(L_2,t)=u_{4xx}(0,t)+u_{5xx}(0,t),$$

$$-\frac{\pi}{3}\beta_2(t)-\frac{2\pi}{3}\varphi_4(t)-\frac{2\pi}{3}\varphi_5(t)+$$

$$+\int_0^t\frac{1}{t-\eta}f''\left(\frac{L_2}{(t-\eta)^{\frac{1}{3}}}\right)\varphi_2(\eta)d\eta+$$

$$+\int_0^t\frac{1}{t-\eta}\varphi''\left(\frac{L_2}{(t-\eta)^{\frac{1}{3}}}\right)\alpha_2(\eta)d\eta- \qquad (18^*)$$

$$-\int_0^t\frac{1}{t-\eta}f''\left(-\frac{L_4}{(t-\eta)^{\frac{1}{3}}}\right)\beta_4(\eta)d\eta=$$

$$=F_{4xx}(0,t)+F_{5xx}(0,t)-F_{2xx}(L_2,t),$$

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(18*) can be derived from (18)

$$-\frac{\pi}{3}\beta_2(t)-\frac{2\pi}{3}\varphi_4(t)-\frac{2\pi}{3}\varphi_5(t)+$$
$$+\int_0^t K_{23}\varphi_2(\eta)d\eta+\int_0^t K_{24}\alpha_2(\eta)d\eta-\int_0^t K_{25}\beta_4(\eta)d\eta= \qquad (18^*)$$
$$=F_{4xx}(0,t)+F_{5xx}(0,t)-F_{2xx}(L_2,t),$$

Satisfying the conditions (4) we have:

$$a)\ u_3(L_3,t)=u_4(L_4,t)=u_6(0,t),$$
$$f(0)\beta_3(t)-f(0)\beta_4(t)+$$
$$+\frac{1}{\Gamma\left(\frac{1}{3}\right)}D_{(0,t)}^{2/3}\int_0^t\frac{1}{(t-\eta)^{1/3}}f\left(\frac{L_3}{(t-\eta)^{1/3}}\right)\varphi_3(\eta)d\eta+$$
$$+\frac{1}{\Gamma\left(\frac{1}{3}\right)}D_{(0,t)}^{2/3}\int_0^t\frac{1}{(t-\eta)^{1/3}}\varphi\left(\frac{L_3}{(t-\eta)^{1/3}}\right)\alpha_3(\eta)d\eta-$$
$$-\frac{1}{\Gamma\left(\frac{1}{3}\right)}D_{(0,t)}^{2/3}\int_0^t\frac{1}{(t-\eta)^{1/3}}\varphi\left(\frac{L_4}{(t-\eta)^{1/3}}\right)\alpha_4(\eta)d\eta$$
$$=\frac{1}{\Gamma\left(\frac{1}{3}\right)}D_{(0,t)}^{2/3}\left[F_4(L_4,t)-F_2(L_3,t)\right] \qquad (19)$$

(19*) can be derived from (19)

$$f(0)\beta_3(t)-f(0)\beta_4(t)+$$
$$+\frac{1}{\Gamma\left(\frac{1}{3}\right)}\int_0^t K_{26}\varphi_3(\eta)d\eta+\frac{1}{\Gamma\left(\frac{1}{3}\right)}\int_0^t K_{27}\alpha_3(\eta)d\eta-$$
$$-\frac{1}{\Gamma\left(\frac{1}{3}\right)}\int_0^t K_{28}\varphi_4(\eta)d\eta-\frac{1}{\Gamma\left(\frac{1}{3}\right)}\int_0^t K_{29}\alpha_4(\eta)d\eta= \qquad (19^*)$$
$$=\frac{1}{\Gamma\left(\frac{1}{3}\right)}D_{(0,t)}^{2/3}\left[F_4(L_4,t)-F_2(L_3,t)\right]$$

$$f(0)\beta_3(t)-f(0)\varphi_6(t)-\varphi(0)\psi_6(t)+$$

$$+\frac{1}{\Gamma\left(\frac{1}{3}\right)}D_{(0,t)}^{2/3}\int_0^t\frac{1}{(t-\eta)^{1/3}}f\left(\frac{L_3}{(t-\eta)^{1/3}}\right)\varphi_3(\eta)d\eta+$$

$$+\frac{1}{\Gamma\left(\frac{1}{3}\right)}D_{(0,t)}^{2/3}\int_0^t\frac{1}{(t-\eta)^{1/3}}\varphi\left(\frac{L_3}{(t-\eta)^{1/3}}\right)\alpha_3(\eta)d\eta= \qquad (20)$$

$$=\frac{1}{\Gamma\left(\frac{1}{3}\right)}D_{(0,t)}^{2/3}\left[F_6(0,t)-F_3(L_3,t)\right]$$

(20*) can be derived from (20)

$$f(0)\beta_3(t)-f(0)\varphi_6(t)-\varphi(0)\psi_6(t)+$$

$$+\frac{1}{\Gamma\left(\frac{1}{3}\right)}\left[\int_0^t K_{30}\varphi_3(\eta)d\eta+\int_0^t K_{31}\alpha_3(\eta)d\eta\right]= \qquad (20^*)$$

$$=\frac{1}{\Gamma\left(\frac{1}{3}\right)}D_{(0,t)}^{2/3}\left[F_6(0,t)-F_3(L_3,t)\right]$$

$$b)\ u_{3x}(L_3,t)=\frac{1}{b_4}u_{4x}(L_4,t)+\frac{1}{b_6}u_{6x}(0,t),$$

$$f'(0)\beta_3(t)-\frac{1}{b_4}f'(0)\beta_4(t)-\frac{1}{b_6}f'(0)\varphi_6(t)-\frac{1}{b_6}\varphi'(0)\psi_6(t)+$$

$$+\frac{1}{\Gamma\left(\frac{2}{3}\right)}D_{(0,t)}^{1/3}\int_0^t\frac{1}{(t-\eta)^{2/3}}f'\left(\frac{L_3}{(t-\eta)^{1/3}}\right)\varphi_3(\eta)d\eta+$$

$$+\frac{1}{\Gamma\left(\frac{2}{3}\right)}D_{(0,t)}^{1/3}\int_0^t\frac{1}{(t-\eta)^{2/3}}\varphi'\left(\frac{L_3}{(t-\eta)^{1/3}}\right)\alpha_3(\eta)d\eta-$$

$$-\frac{1}{b_4\Gamma\left(\frac{2}{3}\right)}D_{(0,t)}^{1/3}\int_0^t\frac{1}{(t-\eta)^{2/3}}f'\left(\frac{L_4}{(t-\eta)^{1/3}}\right)\varphi_4(\eta)d\eta-$$

$$-\frac{1}{b_4\Gamma\left(\frac{2}{3}\right)}D_{(0,t)}^{1/3}\int_0^t\frac{1}{(t-\eta)^{2/3}}\varphi'\left(\frac{L_4}{(t-\eta)^{1/3}}\right)\alpha_4(\eta)d\eta= \qquad (21)$$

$$=\frac{1}{\Gamma\left(\frac{2}{3}\right)}D_{(0,t)}^{1/3}\left[\frac{1}{b_4}F_{4x}(L_4,t)+\frac{1}{b_6}F_{6x}(0,t)-F_{3x}(L_3,t)\right]$$

(21*) can be derived from (21)

$$f'(0)\beta_3(t)-\frac{1}{b_4}f'(0)\beta_4(t)-\frac{1}{b_6}f'(0)\varphi_6(t)-\frac{1}{b_6}\varphi'(0)\psi_6(t)$$

$$+\frac{1}{\Gamma\left(\frac{2}{3}\right)}\left[\int_0^t K_{32}\varphi_3(\eta)d\eta+\int_0^t K_{33}\alpha_3(\eta)d\eta\right]+$$

$$+\frac{1}{\Gamma\left(\frac{2}{3}\right)}\left[\int_0^t K_{34}\varphi_4(\eta)d\eta-\int_0^t K_{35}\alpha_4(\eta)d\eta\right]= \qquad (21^*)$$

$$=\frac{1}{\Gamma\left(\frac{2}{3}\right)}D_{(0,t)}^{1/3}\left[\frac{1}{b_4}F_{4x}(L_4,t)+\frac{1}{b_6}F_{6x}(0,t)-F_{3x}(L_3,t)\right]$$

$c)\ u_{3xx}(L_3,t)=u_{4xx}(L_4,t)+u_{6xx}(0,t),$

$$-\frac{\pi}{3}\beta_3(t)-\frac{2\pi}{3}\beta_4(t)-\frac{2\pi}{3}\varphi_6(t)+$$

$$+\int_0^t\frac{1}{(t-\eta)}f''\left(\frac{L_3}{(t-\eta)^{1/3}}\right)\varphi_3(\eta)d\eta d\eta- \qquad (22)$$

$$-\int_0^t\frac{1}{(t-\eta)}\varphi''\left(\frac{L_4}{(t-\eta)^{1/3}}\right)\alpha_4(\eta)d\eta=$$

$$=\left[F_{4xx}(L_4,t)+F_{6xx}(0,t)-F_{3xx}(L_3,t)\right]$$

(22*) can be derived from (22)

$$-\frac{\pi}{3}\beta_3(t)-\frac{2\pi}{3}\beta_4(t)-\frac{2\pi}{3}\varphi_6(t)+$$

$$+\int_0^t K_{36}\varphi_3(\eta)d\eta+\int_0^t K_{37}\alpha_3(\eta)d\eta-\int_0^t K_{38}\varphi_4(\eta)d\eta-\int_0^t K_{39}\alpha_4(\eta)d\eta= \qquad (22^*)$$

$$=\left[F_{4xx}(L_4,t)+F_{6xx}(0,t)-F_{3xx}(L_3,t)\right]$$

where the kernels of integral operators defined as:

$$K_1=\int_\eta^t \frac{1}{(t-\tau)^{2/3}(\tau-\eta)^{1/3}} f'\left(\frac{L_1}{(\tau-\eta)^{1/3}}\right)d\tau\,,\; K_2=\int_\eta^t \frac{1}{(t-\tau)^{2/3}(\tau-\eta)^{1/3}} f'\left(-\frac{L_2}{(\tau-\eta)^{1/3}}\right)d\tau$$

$$K_3=\int_\eta^t \frac{1}{(t-\tau)^{2/3}(\tau-\eta)^{1/3}} f'\left(\frac{L_1}{(\tau-\eta)^{1/3}}\right)d\tau\,, K_4=\int_\eta^t \frac{1}{(t-\tau)^{2/3}(\tau-\eta)^{1/3}} f'\left(-\frac{L_3}{(\tau-\eta)^{1/3}}\right)d\tau$$

$$K_5=\int_\eta^t \frac{1}{(t-\tau)^{1/3}(\tau-\eta)^{2/3}} f''\left(\frac{L_1}{(\tau-\eta)^{1/3}}\right)d\tau ,$$

$$K_6=\int_\eta^t \frac{1}{(t-\tau)^{1/3}(\tau-\eta)^{2/3}} f''\left(-\frac{L_2}{(\tau-\eta)^{1/3}}\right)d\tau$$

$$K_7=\int_\eta^t \frac{1}{(t-\tau)^{1/3}(\tau-\eta)^{2/3}} f''\left(\frac{L_1}{(\tau-\eta)^{1/3}}\right)d\tau\,, K_8=\int_\eta^t \frac{1}{(t-\tau)^{1/3}(\tau-\eta)^{2/3}} f''\left(-\frac{L_3}{(\tau-\eta)^{1/3}}\right)d\tau$$

,

$$K_9=\int_0^x U(y+L_2;t-\eta)dy\,,\; K_{10}=\int_0^x U(y+L_3;t-\eta)dy\,,\; K_{11}=\int_0^x U(y-L_1;t-\eta)dy$$

$$K_{12}=\int_\eta^t \frac{1}{(t-\tau)^{2/3}(\tau-\eta)^{1/3}} f'\left(-\frac{L_1}{(\tau-\eta)^{1/3}}\right)d\tau ,$$

$$K_{13}=\int_\eta^t \frac{1}{(t-\tau)^{2/3}(\tau-\eta)^{1/3}} f'\left(\frac{L_2}{(\tau-\eta)^{1/3}}\right)d\tau ,$$

$$K_{14}=\int_\eta^t \frac{1}{(t-\tau)^{2/3}(\tau-\eta)^{1/3}} \varphi'\left(\frac{L_2}{(\tau-\eta)^{1/3}}\right)d\tau ,$$

$$K_{15}=\int_\eta^t \frac{1}{(t-\tau)^{2/3}(\tau-\eta)^{1/3}} f'\left(-\frac{L_2}{(\tau-\eta)^{1/3}}\right)d\tau ,$$

$$K_{16}=\int_\eta^t \frac{1}{(t-\tau)^{2/3}(\tau-\eta)^{1/3}} f'\left(\frac{L_2}{(\tau-\eta)^{1/3}}\right)d\tau\,,\; K_{17}=\int_\eta^t \frac{1}{(t-\tau)^{2/3}(\tau-\eta)^{1/3}} \varphi'\left(\frac{L_2}{(\tau-\eta)^{1/3}}\right)d\tau ,$$

$$K_{18}=\int_\eta^t \frac{1}{(t-\tau)^{1/3}(\tau-\eta)^{2/3}} f''\left(\frac{L_2}{(\tau-\eta)^{1/3}}\right)d\tau\,,\; K_{19}=\int_\eta^t \frac{1}{(t-\tau)^{1/3}(\tau-\eta)^{2/3}} \varphi''\left(\frac{L_2}{(\tau-\eta)^{1/3}}\right)d\tau$$

, $$K_{20}=\int_{\eta}^{t}\frac{1}{(t-\tau)^{1/3}(\tau-\eta)^{2/3}}f''\left(-\frac{L_4}{(\tau-\eta)^{1/3}}\right)d\tau$$,

$$K_{21}=\int_{\eta}^{t}\frac{1}{(t-\tau)^{2/3}(\tau-\eta)^{1/3}}\varphi'\left(\frac{L_2}{(\tau-\eta)^{1/3}}\right)d\tau\ ,\ K_{22}=\int_{\eta}^{t}\frac{1}{(t-\tau)^{1/3}(\tau-\eta)^{2/3}}f''\left(\frac{L_2}{(\tau-\eta)^{1/3}}\right)d\tau$$

, $$K_{23}=\int_{0}^{x}U(y-L_2;t-\eta)dy$$

$$K_{24}=\int_{0}^{x}V(y-L_2;t-\eta)dy\ ,\ K_{25}=\int_{0}^{x}U(y-L_4;t-\eta)dy\ ,$$

$$K_{26}=\int_{\eta}^{t}\frac{1}{(t-\tau)^{2/3}(\tau-\eta)^{1/3}}f'\left(\frac{L_3}{(\tau-\eta)^{1/3}}\right)d\tau\ ,K_{27}=\int_{\eta}^{t}\frac{1}{(t-\tau)^{2/3}(\tau-\eta)^{1/3}}\varphi'\left(\frac{L_3}{(\tau-\eta)^{1/3}}\right)d\tau\ ,$$

$$K_{28}=\int_{\eta}^{t}\frac{1}{(t-\tau)^{2/3}(\tau-\eta)^{1/3}}f'\left(\frac{L_4}{(\tau-\eta)^{1/3}}\right)d\tau\ ,K_{29}=\int_{\eta}^{t}\frac{1}{(t-\tau)^{2/3}(\tau-\eta)^{1/3}}\varphi'\left(\frac{L_4}{(\tau-\eta)^{1/3}}\right)d\tau\ ,$$

$$K_{30}=\int_{\eta}^{t}\frac{1}{(t-\tau)^{2/3}(\tau-\eta)^{1/3}}f'\left(\frac{L_3}{(\tau-\eta)^{1/3}}\right)d\tau\ ,\ K_{31}=\int_{\eta}^{t}\frac{1}{(t-\tau)^{2/3}(\tau-\eta)^{1/3}}\varphi'\left(\frac{L_3}{(\tau-\eta)^{1/3}}\right)d\tau$$

$$K_{32}=\int_{\eta}^{t}\frac{1}{(t-\tau)^{1/3}(\tau-\eta)^{2/3}}f''\left(\frac{L_3}{(\tau-\eta)^{1/3}}\right)d\tau\ ,\ K_{33}=\int_{\eta}^{t}\frac{1}{(t-\tau)^{1/3}(\tau-\eta)^{2/3}}\varphi''\left(\frac{L_3}{(\tau-\eta)^{1/3}}\right)d\tau$$

$$K_{34}=\int_{\eta}^{t}\frac{1}{(t-\tau)^{1/3}(\tau-\eta)^{2/3}}f''\left(\frac{L_4}{(\tau-\eta)^{1/3}}\right)d\tau\ ,\ K_{35}=\int_{\eta}^{t}\frac{1}{(t-\tau)^{1/3}(\tau-\eta)^{2/3}}\varphi''\left(\frac{L_4}{(\tau-\eta)^{1/3}}\right)d\tau$$

$$K_{36}=\int_{0}^{x}U(y-L_3;t-\eta)dy\ ,K_{37}=\int_{0}^{x}V(y-L_3;t-\eta)dy\ ,$$

$$K_{38}=\int_{0}^{x}U(y-L_4;t-\eta)dy\ ,K_{39}=\int_{0}^{x}V(y-L_4;t-\eta)dy\ ,$$

We obtained the system of integral equations (8) – (22) with respect to unknowns

$$\Phi(t)=(\varphi_k(t),\psi_n(t),\alpha_i(t),\beta_i(t))^T.\ \ k=\overline{1,7};\ n=1,2;i=2,3,4$$

$$\text{matrix } A = \begin{pmatrix} A_1 & 0 & 0 \\ 0 & A_2 & 0 \\ 0 & 0 & A_3 \end{pmatrix}$$

$$\text{I-Bondmatrix}: A_1 = \begin{pmatrix} f(0) & -f(0) & 0 & -\varphi(0) & 0 & 0 \\ f(0) & 0 & -f(0) & 0 & -\varphi(0) & 0 \\ f'(0) & -\frac{1}{b_2}f'(0) & 0 & -\frac{1}{b_2}\varphi'(0) & 0 & 0 \\ f'(0) & 0 & -\frac{1}{b_3}f'(0) & 0 & -\frac{1}{b_2}\varphi'(0) & 0 \\ -\frac{\pi}{3} & -\frac{2\pi}{3} & -\frac{2\pi}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & f(0) \end{pmatrix}$$

$$\det A_1 \neq 0$$

$$\text{II-Bondmatrix}: A_2 = \begin{pmatrix} f(0) & 0 & -\varphi(0) & f(0) & 0 \\ 0 & -f(0) & 0 & f(0) & -\varphi(0) \\ -\frac{1}{b_4}f'(0) & 0 & -\frac{1}{b_4}\varphi'(0) & f'(0) & 0 \\ 0 & -\frac{1}{b_5}f'(0) & 0 & f'(0) & -\frac{1}{b_5}\varphi'(0) \\ -\frac{2\pi}{3} & -\frac{2\pi}{3} & 0 & -\frac{\pi}{3} & 0 \end{pmatrix}$$

$$\det A_2 \neq 0$$

III-Bondmatrix: $$A_3 = \begin{pmatrix} 0 & f(0) & -f(0) & 0 \\ -f(0) & f(0) & 0 & -\varphi(0) \\ -\frac{1}{b_6} f'(0) & f'(0) & -\frac{1}{b_4} f'(0) & -\frac{1}{b_6}\varphi'(0) \\ -\frac{2\pi}{3} & -\frac{\pi}{3} & -\frac{2\pi}{3} & 0 \end{pmatrix}$$

$$\det A_3 \neq 0$$

$$\det A \neq 0$$

According to the asymptotes of Airy functions the kernels of the integral operators are integrals (see [14, 15]). Hence, it follows from the uniqueness theorem and Fredholm alternatives that the system of equations has a unique solution. Thus the solvability of the problem is proved.

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