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SOLVING THE BOUNDARY PROBLEM BY THE METHOD OF GREEN'S FUNCTION FOR THE SIMPLE DIFFERENTIAL EQUATION OF THE SECOND ORDER LINEAR

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ABSTRACT

Green functions are used mainly to solve certain types of linear inhomogeneous partial differential equations, although homogeneous partial differential equations can also be solved using this approach. In this article is discussed about how to solve or how to find solution of differential equations using Green's function. Furthermore, after theory, some examples are given to explain.

KEYWORDS: *Green's Function, Differential Equation, Boundary Problem, Algebraic System.*

INTRODUCTION

In mathematics, a Green's function is the impulse response of an inhomogeneous linear differential operator defined on a domain with specified initial conditions or boundary conditions. Green's functions are named after the British mathematician George Green, who first developed the concept in the 1820s. in the modern study of linear partial differential equations, Green's functions are studied largely from the point of view of fundamental solutions instead.

The Green's function as used in physics is usually defined with the opposite sign, instead. That is,

$$L\ G(x,s) = \delta(x-s).$$

This definition does not significantly change any of the properties of the Green's function due to the evenness of the Dirac delta function.

Sometimes the Green's function can be split into a sum of two functions. One with the variable positive $(+)$ and the other with the variable negative $(-)$. These are the advanced and retarded Green's function, and when the equation under study depends on time, one of the parts is causal and the other anti-causal. In these problems usually the causal part is the important one. These are frequently the solutions to the inhomogeneous electromagnetic wave equation.

A Green's function is a solution to an inhomogeneous differential equation with a " driving term" that is a delta function. It provides a convenient method for solving more complicated inhomogeneous differential equation. In physics, Green's function methods are used to describe a wide range of physical phenomena, such as the response of mechanical systems to impact or the emission of sound waves from acoustic sources.

Let it be reqiured the creation of a Green's function that satisfies the following

$$a(x)y''+b(x)y'+c(x)y=f(x),\ x\in[a,b] \qquad (1)$$

differential equation and

$$\begin{cases}\alpha y(a)+\beta y'(a)=0\\ \gamma y(b)+\delta y'(b)=0\end{cases} \qquad (2)$$

Boundary problem.

In this case, the Green's function of the boundary problem of (1), (2) is called a $G(x,s)$, $\forall x\in[a,b]$, $S\in(0,b)$ continuous function that if these conditions are satisfied,

1°. When $x\neq s$, $G(x,s)$ function is satisfied the

$$a(x)y''+b(x)y'+c(x)y=0 \qquad (3)$$

equation.

2°. When $x=a$ and $x=b$, $G(x,s)$ function is satisfied the boundary problem of (2).

3°. When $x=s$, $G(x,s)$ function continuous on *x*, its derivative $G'_x(x,s)$ has a finite interruption at $x=s$ point, that is, its jump is equal to $\frac{1}{q(s)}$.

Namely,

$$G(s+0,s)=G(s-0,s)$$

$$G'_x(s+0,s)-G'_x(s-0,s)=\frac{1}{q(s)} \qquad (4)$$

to determine the Green's function corresponding for the boundary problem, firstly, it is necessary to find a two linear free solution of the homogeneous equation (3). They should be satisfied the boundary conditions (2) accordingly. In case, the Grenn's function will exist and it will be searched in the form

$$G(x,s)=\begin{cases}\varphi(s)y_1(x), & a\le x\le s\\ \psi(s)y_2(x), & s\le x\le b\end{cases}.$$

$\varphi(s)$, $\psi(s)$ are functions of (3), we derive them from the (4) equality.

From this algebraic system

$$\varphi(s)y_1(x)-\psi(s)y_2(x)=0$$

$$\varphi(s)y'_1(x)+\psi(s)y'_2(x)=\frac{1}{q(s)}$$

When the Green's function is present, the formula $y(x)=\int_a^b G(x,s)f(s)ds$ will be solution of the boundary problem of (1), (2) and here

$$y(x)=y_1(x)\int_x^b \frac{y_2(s)f(s)}{w(s)}ds+y_2(x)\int_a^x \frac{y_1(s)f(s)}{w(s)}dx .$$

Look at the the following problem.

Problem. Create the Green's function. $y''-y=f(x)$ let all $x\in(-\infty;+\infty)$ be limited at $y(x)$.

Solution. $L(y)=y''-y=0$ the special solutions of the equation $y_1(x)=e^x$ and $y_2(x)=e^{-x}$ free linear, general solution

$$y=c_1e^x+c_2e^{-x} .$$

$x\to-\infty$ first special solution $y_1(x)=e^x$ will be limeted. $y_2(x)=e^{-x}$ is limited at $x\to+\infty$. We look for the Green's function in the following view:

$$G(x,s)=\begin{cases}\varphi(s)e^s & -\infty<x\le s\\ \psi(s)e^{-s} & s\le x<+\infty.\end{cases}$$

Here, we choose $\varphi(s)$ and $\psi(s)$ functions so that satisfy the following equality:

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$$G(s+0,s)=G(s-0,s)$$

$$G'_x(s+0,s)-G'_x(s-0,s)=\frac{1}{q(s)}.$$

Here $q(s)=1$, coefficient of y'' .

$$\begin{cases}\psi(s)e^{-s}=\varphi(s)e^{s}\\ -\psi(s)e^{-s}=\varphi(s)e^{s}+1\end{cases}$$

from that

$$\varphi(s)=\frac{1}{2}e^{-s},\ \psi(s)=-\frac{1}{2}e^{s}$$

Answer: $$G(x,s)=\begin{cases}-\frac{1}{2}e^{x-s} & -\infty<x\le s\\ -\frac{1}{2}e^{s-x} & s\le x<+\infty\ .\end{cases}$$

CONCLUSION

The Green's function integral equation method is a method for solving linear differential equations by the expressing the solution in terms of an integral equation, where the integral involves an overlap integral between the solution itself and Green's function. Green's functions are widely used in electrodynamics and quantum field theory, where the relevant differential operators are often difficult or impossible to solve exactly but can be solved perturbatively using Green's function is often called the propagator or two-point correlation function since it is related to the probability of measuring a field at one point given that it is soursed at a different point.

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