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CRS RESULTS IN DMUS – DEA

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ABSTRACT

Data Envelopment Analysis (DEA) is a nonparametric method in operations research and economics for the estimation of production frontiers. It is used to empirically measure productive efficiency of Decision Making Units (DMUs). The DEA is a mathematical programming technique that finds number of practical applications to measure the performance of similar units.DEA is a methodology based upon an interesting application of linear programming technique and it was originally developed for performance measurement. The present research study is to measure the CRS results in DMUs for random data.

KEYWORDS: *CRS (Constant Returns to Scale), DEA (Data Envelopment Analysis), DMUs, Efficiency.*

I. INTRODUCTION

In judging the performance of a production unit, one commonly examines whether or not the unit is productive efficient. The Data Envelopment Analysis (DEA) is a linear programming based technique for measuring the performance efficiency of organizational units which are termed as Decision Making Units (DMUs). This technique aims to measure how efficiently a decision making unit uses the resources available to generate a set of outputs. This method has been successfully employed for assessing the relative performance of set of firms that uses a variety of identical inputs to produce a variety of identical outputs.

Generally, the performance of a DMU is assessed with DEA and is obtained by using the concept of efficiency which is the ratio of weighted sum of outputs to a weighted sum of inputs. Efficiencies obtained by using DEA are relative to the best performance of a virtual DMU. The best performing DMU is assigned an efficiency score of unity and the performance of others varies between zero and one.

The DEA is a mathematical programming technique that finds number of practical applications to measure the performance of similar units, such as a set of hospitals, a set of schools, a set of industries etc. Thus, DEA is a methodology based upon an interesting application of linear programming technique and it was originally developed for performance measurement.

II.METHODOLOGY

Data envelopment analysis is a deterministic approach employed to measure input and output technical efficiencies. In a firm or production unit inputs are combined to produce one or more outputs subject to technology. The techniques of production vary from one unit to another. This kind of variation causes efficiency differences among the competing decision making units. Efficient measurement dates back to Farrell whose pioneering work sparked off interest in several researchers in producer's theory.

Assume there is data on K inputs and M outputs on each of N firms or DMUs as they tend to be called in the DEA literature. For the i^{th} DMU these are represented by the vectors x_i and Y_i respectively. The K x N input matrix X, and the M x N output matrix Y, represent the data of all N DMUs. The purpose of DEA is to construct a non-parametric envelopment frontier over the data points such that all observed points lie on or below the production frontier. For the simple example of an industry where one output is produced using two inputs, it can be visualized as a number of intersecting planes forming a tight fitting cover over a scatter of points in three-dimensional space. Given the CRS assumption this can also be represented by a unit isoquant in input/output space.

The best way to introduce DEA is via the ratio form. For each DMU we would like to obtain a measure of the ratio of all outputs over all inputs, such as $u'y_i/v'x_i$, where u is an Mx1 vector of output weights and v is a Kx1 vector of input weights. To select optimal weights we specify the mathematical programming problem:

$$\begin{aligned} &\mathrm{Max}_{u,v}\,(u'y_i/v'x_i) \\ \text{Subject to}\quad &u'y_j/v'x_j \le 1,\ j=1,2,\ldots.N. \qquad \ldots(2.2.1) \\ &u, v \ge 0. \end{aligned}$$

This involves finding values for u and v, such that the efficiency measure of the i^{th} DMU is maximized, subject to the constraint that all efficiency measures must be less than or equal to one. One problem with this particular ratio formulation is that it has an infinite number of solutions. To avoid this one can impose the constraint $v'x_i=1$, which provides:

$$\begin{aligned} &\mathrm{Max}_{\mu,v}\,(\mu'y_i) \\ \text{Subject to}\quad &v'x_i = 1, \\ &\mu'y_j - v'x_j \le 0,\ j=1,2,\ldots.N, \qquad \ldots(2.2.2) \\ &\mu, v \ge 0. \end{aligned}$$

Where the notation change from u and v to μ and v reflects the transformation. This form is known as the multiplier form of the linear programming problem.

Using the duality in linear programming, one can derive an equivalent envelopment form of this problem:

$$\text{Min}_{\theta,\lambda}\ \theta$$

$$\text{Subject to} \quad -y_i + Y\lambda \geq 0, \qquad \ldots(2.2.3)$$
$$\theta x_i - X\lambda \geq 0,$$
$$\lambda \geq 0.$$

where θ is a scalar and λ is a N x l vector of constants. This envelopment form involves fewer constraints than the multiplier form (K+M < N+1), and hence is generally the preferred form to solve. The value of θ obtained will be the efficiency score for the i^{th} DMU. It will satisfy $\theta \leq 1$, with a value of 1 indicating a point on the frontier and hence a technically efficient DMU, according to the Farrell (1957) definition

III. EMPIRICAL INVESTIGATION

Input orientated DEA

Scale assumption: CRS

Slacks calculated using multi-stage method

EFFICIENCY SUMMARY:

firm	Te
1	0.500
2	1.000
3	0.833
4	0.714
5	1.000
Mean	0.810

SUMMARY OF OUTPUT SLACKS:

Firm output:	1
1	0.000
2	0.000
3	0.000
4	0.000
5	0.000
Mean	0.000

SUMMARY OF INPUT SLACKS:

Firm input:	1	2
1	0.000	0.500
2	0.000	0.000
3	0.000	0.000
4	0.000	0.000
5	0.000	0.000
Mean	0.000	0.100

SUMMARY OF PEERS:

Firm peers:

1	2	
2	2	
3	5	2
4	5	2
5	5	

SUMMARY OF PEER WEIGHTS: (in same order as above)

Firm peer weights:

1	0.500	
2	1.000	
3	0.500	1.000
4	0.286	0.214
5	1.000	

PEER COUNT SUMMARY:

(i.e., no. times each firm is a peer for another)

Firm	peer count:
1	0
2	3
3	0
4	0
5	2

SUMMARY OF OUTPUT TARGETS:

Firm output: 1

1	1.000
2	2.000
3	3.000
4	1.000
5	2.000

SUMMARY OF INPUT TARGETS:

Firm input:	1	2
1	1.000	2.000
2	2.000	4.000
3	5.000	5.000
4	2.143	1.429
5	6.000	2.000

FIRM BY FIRM RESULTS:

Results for firm: 1

Technical efficiency = 0.500

PROJECTION SUMMARY:

variable		original value	radial movement	slack movement	projected value
output	1	1.000	0.000	0.000	1.000
input	1	2.000	-1.000	0.000	1.000
input	2	5.000	-2.500	-0.500	2.000

LISTING OF PEERS:

peer	lambda weight
2	0.500

Results for firm: 2

Technical efficiency = 1.000

PROJECTION SUMMARY:

variable	original value	radial movement	slack movement	projected value

output	**1**	**2.000**	**0.000**	**0.000**	**2.000**
input	**1**	**2.000**	**0.000**	**0.000**	**2.000**
input	**2**	**4.000**	**0.000**	**0.000**	**4.000**

LISTING OF PEERS:

peer	**lambda weight**
2	**1.000**

Results for firm: 3

Technical efficiency = 0.833

PROJECTION SUMMARY:

Variable		**original Value**	**radial movement**	**slack movement**	**projected value**
output	**1**	**3.000**	**0.000**	**0.000**	**3.000**
input	**1**	**6.000**	**-1.000**	**0.000**	**5.000**
input	**2**	**6.000**	**-1.000**	**0.000**	**5.000**

LISTING OF PEERS:

peer	**lambda weight**
5	**0.500**
2	**1.000**

Results for firm: 4

Technical efficiency = 0.714

PROJECTION SUMMARY:

variable		**original value**	**radial movement**	**slack movement**	**projected value**
output	**1**	**1.000**	**0.000**	**0.000**	**1.000**
input	**1**	**3.000**	**-0.857**	**0.000**	**2.143**
input	**2**	**2.000**	**-0.571**	**0.000**	**1.429**

LISTING OF PEERS:

peer	**lambda weight**
5	**0.286**
2	**0.214**

Results for firm: 5

Technical efficiency = 1.000

PROJECTION SUMMARY:

Variable		original value	radial movement	slack movement	projected value
Output	**1**	**2.000**	**0.000**	**0.000**	**2.000**
Input	**1**	**6.000**	**0.000**	**0.000**	**6.000**
input	**2**	**2.000**	**0.000**	**0.000**	**2.000**

LISTING OF PEERS:

peer	lambda weight
5	**1.000**

IV.CONCLUSION:

The present study aims at constructed and solved linear programming problems to measure the CRS results in DMUs (firms).

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