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# ASSESMENT OF METROLOGICAL RELIABILITY OF MEASUREMENTS USING THE METHOD OF PRODUCING FUNCTIONS

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## ABSTRACT

*In the suggested article, a confidence indicator is proposed as an evaluation of metrological reliability. The quantitative value of confidence indicator can be estimated by means of the method of generating functions. This is a scientific novelty of the work. Relevance of the problem of assessing the measuring instruments metrological reliability evaluation is substantiated in this paper since the current trend towards structural and functional complexity of measuring instruments may lead to decreasing of their reliability and, in particular, metrological reliability. The main goal of this work is to systematize the problems of reliability of measuring instruments and evaluate their metrological reliability using the method of generating functions.*

**KEYWORDS:** *Reliability Indicator, Metrological Reliability, Assessment, Measuring Instruments.*

## INTRODUCTION

Assessment of the reliability of various technical means, including measuring instruments (MI) of physical quantities, has been and remains one of the urgent problems facing the developers of such devices and those who operate them. The urgency of this problem is increasing, since the current trend aimed at structural and functional complication of technical devices can lead to a decrease in their reliability. The solution to this problem largely depends on both the preliminary calculation of reliability in the process of developing a technical device or system in order to determine the predicted reliability characteristics, and their periodic assessment during operation.

In accordance with the interstate standard for reliability in engineering, reliability is understood as the property of an object to retain over time the ability to perform the required functions in specified modes and conditions of use, maintenance, storage and transportation. In this case, the reliability indicators are stability, reliability, durability, maintainability and preservation [1].

**MATERIALS AND METHODS**

In reliability theory, it is customary to distinguish between functional and metrological reliability. The functional reliability of a technical device is determined by functional (sudden) failures, which are of an obvious nature, appear suddenly and can be detected without carrying out its verification.

Functional failures lead to the termination of the operability of the technical device, the system as a whole or their individual units. If we are talking about SI of physical quantities, then sudden failures can also occur in them, characterized by an abrupt change in one or more metrological characteristics (MCh), that is, characteristics that determine the main purpose of the MI. These failures, due to their random manifestation, cannot be predicted. The consequences of these failures, for example, a sudden failure of power supplies, measuring sensors or loss of their sensitivity, etc., are easily detected during the operation of the measuring device and are obvious. A feature of such failures is the constancy of their intensity over time, which makes it possible to apply the classical theory of reliability for their analysis. Other characteristics that determine the functional reliability of the MI are also related to the failure rate of the MI: the probability of failure-free operation of the MI and the mean time of failure-free operation (mean time between failures). Since an accidental failure can occur at any time, regardless of how long the SI has worked, the intensity of sudden failures does not depend on time; accordingly, formulas and algorithms for finding the probabilities of no-failure operation and the mean time of no-failure operation of the SI are greatly simplified.

Metrological reliability refers primarily to the characteristics of the MI and is defined as the probability of keeping the MCh of MI within the range of tolerances for these characteristics for a certain time interval. It should also be noted here that if the issues of assessing the functional reliability of measuring instruments are well covered in domestic and foreign literature, then the issues related to the assessment of metrological reliability are covered very sparingly. This is due to the complexity of the algorithm for assessing the metrological reliability of the measuring instrument, as well as the fact that at present there is no universal mathematical model of the measuring instrument's metrological failures. Metrological reliability of measuring instruments is associated with such a concept as metrological characteristics. In accordance with the interstate standard "Normalized metrological characteristics of measuring instruments" [2], metrological characteristics are those characteristics that are intended to assess the technical level and quality of measurements, to determine the measurement results and to estimate the characteristics of the instrumental measurement error. The standard establishes a set of standardized characteristics of the MCh of MI, which are divided into groups: characteristics intended for determining the measurement results; characteristics of MI errors; characteristics of the MI sensitivity to influencing quantities; dynamic characteristics of MI; uninformative parameters of the MI output signal.

Metrological reliability is closely related to such a concept as metrological failure, which is identified as the exit of the MCh of MI from the established permissible limits. As the studies have shown, metrological failures of measuring instruments occur much more often than functional failures, which necessitates the development of special methods for their prediction and detection [3].

**Results**

The article proposes a method for assessing the metrological reliability of SI using the method of generating functions, which allows, on the basis of the selected mathematical model of the evolution of the SI error and the proposed metrological reliability indicator, to carry out the metrological forecast of the SI. As a model for the evolution of the SI error, a gradual failure model with a discrete change in the error in time was chosen, which is characteristic of a certain class of SI, for example, time intervals meters, etc. The paths of evolution (change) of errors in such a model are shown in Fig. 1.

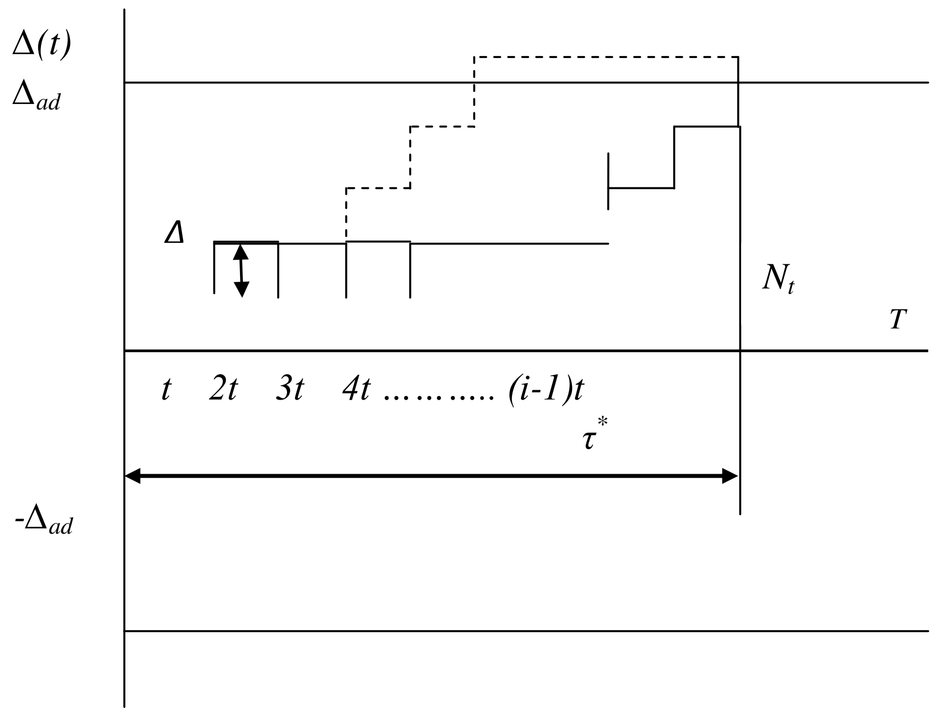


Fig. 1. Trajectories of MI error evolution with discrete changes in absolute errors in time.

The metrological reliability indicator for the specified SI class was chosen the reliability indicator, which in this case acts as a measure of metrological reliability and is defined as the probability of maintaining the SI error within the tolerance range for these characteristics over the measurement interval. The confidence indicator can be written in the following form:

$$D = P\{|\Delta(t)| \leq \Delta_{ad}\}_{t \in \tau_*}, \quad (1)$$

where$P$ is the probability of metrological failure;

$\Delta(t)$ is the absolute SI error characterizing the process of evolution of the error in time $t$;

$\Delta_{ad}$ - area of admittance;

$\tau^*$ - measurement time interval.

It is known from the theory of Markov chains that the probability $P$ is finite and can be appropriately determined. In fig. 1 shows the possible (we will restrict ourselves to two realizations) trajectories of the evolution of the error for the model of gradual failure with a discrete change in the error in time. The intersection of the trajectories of the boundaries of the permissible absolute error $\pm\Delta_{ad}$is pre-identified as a metrological failure.

Taking into account expression (1), the reliability indicator can be determined from the ratio.

$$D = 1 - (P^- + P^+), \qquad (2)$$

where, $P^-$, $P^+$ - the probabilities of reaching or exceeding the absolute error, respectively, of the lower and upper limits of the admittance field for the time $\tau^*$.

The probabilities of metrological failures $P^-$ and $P^+$can be represented as the following sums:

$$P^- = \sum_{h=1}^{N_t} U_{x_0,h};\ P^+ = \sum_{h=1}^{N_t} V_{x_0,h}, \qquad (3)$$

where$U_{x_0,h}$ and $V_{x_0,h}$ are the probabilities of reaching at the $h$-th step, respectively, the lower and upper boundaries of the admittance field;

$N_t = {\tau^*}/{t}$ - the interval of storage of the accumulated error within the admittance field $\pm\Delta_{ad}$ add, expressed through the parameter $t$.

The expression for the confidence indicator can be represented as

$$D = 1 - \sum_{h=1}^{N_t} \quad (U_{x_0,h} + V_{x_0,h}), \qquad (4)$$

The probabilities $U_{x_0,h}$ and $V_{x_0,h}$ can be determined from the difference equations of two variables $x_0$ and $h$ of the form

$$U_{x_0,h+1} = P_+ U_{x_0+1,h} + P_- U_{x_0-1,h}; \qquad (5)$$

$$V_{k_t,h+1} = P_+ U_{k_t-x_0-1,h} + P_- U_{k_t-x_0+1,h}, \qquad (6)$$

Where $k_t = \frac{2(\Delta_{ad})}{\Delta}$ is the value of the admittance field, expressed in increments of the value $\Delta$;

$x_0 = \frac{\Delta_0}{\Delta}$the initial value of the error, expressed in increments of the value $\Delta$;

$P_+$- the probability of the "positive" error increment by the value $\Delta$;

$P_-$ - the probability of an increment of the "negative" error by the value $\Delta$.

To solve equations (5) and (6), we will use the method of generating functions, which will significantly simplify the solutions of these equations and then apply a more efficient computational algorithm to find the reliability indicator. The method of generating functions was first proposed by the outstanding scientist L. Euler. The method has been and is being applied to compactly record information about various sequences, find dependencies for a sequence of numbers given by a recurrent relation (for example, for Fibonacci numbers), study the asymptotic behavior of sequences, represent discrete distribution laws of a discrete random variable and compositions of distribution laws, calculate moments of discrete laws a random variable, etc. The fairly widespread use of this method is due to the possibility of constructing on its basis effective computational algorithms for solving certain problems, which is relevant at the current level of development of information technologies. In general, the generating function is a formal power series of the form:

$$\varphi(z) = \sum_{k=0}^{\infty} P_k z^k, \qquad (7)$$

Generating (producing) the sequence of coefficients $P_0$, $P_1$, $P_2$…, with $\sum_{k=0}^{\infty} P_k = 1; 0 \leq z \leq 1.$

The term "formal" means that for a given series, the region of convergence of the series is not defined. Each coefficient $P_k$ is numerically equal to the probability of occurrence of such a number of events, the number of which is equal to the exponent z. We indicate here the main properties of generating functions:

$\varphi(z), \varphi'(z)$ … the rows converge absolutely;

$\varphi(z) = \sum_{k=0}^{\infty} P_k z^k ; \varphi(1) = 1; \varphi(1) = \sum_{k=0}^{\infty} P_k = 1;$

$$\varphi'(z) = \sum_{k=0}^{\infty} P_k k z^{k-1}; \ \varphi'(z) = \sum_{k=0}^{\infty} P_k\, k = m_1$$

$$\varphi''(z) = \sum_{k=0}^{\infty} P_k k(k-1) z^{k-2}; \ \varphi'^{(1)} = \sum_{k=0}^{\infty} P_k\, k(k-1) = \sum_{k=0}^{\infty} P_k\, k^2 - \sum_{k=0}^{\infty} P_k\, k = m_2 - m_1$$

$\varphi'(1) + \varphi''(1) = m_2.$

Using the method of generating functions, we represent the generating function in the form

$$V_{x_0}(z) = \sum_{h=0}^{\infty} U_{x_0,h} z^h. \qquad (8)$$

Multiply equation (5) by $z^{h+1}$ and summing it over $h$, we obtain a new difference equation for one parameter $z$:

$$U_{x_0}(z) = P_+ z V_{x_0} + 1(z) + P_- z V_{x_0-1}(z). \qquad (9)$$

For the resulting equation, boundary conditions of the form

$$V_0(z) = 1; \ V_{k_t}(z) = 0.$$

Particular solutions of equation (9) will have the form

$V_{x_0}(z) = \beta^{x_0}(z).$

Substituting particular solutions into equation (9), we obtain a quadratic characteristic equation for finding $\beta(z)$, which has the following roots:

$$\beta_{1,2}(z) = \frac{1 + \sqrt{1 - 4P_+ P_- z^2}}{2P_+ z}.$$

which allows for any arbitrary functions A (z) and B (z) to write the general solution of Eq. (9) in the form

$V_{x_0}(z) = A(z)\beta_1^{x_0}(z) + B(z)\beta_2^{x_0}(z).$

Taking into account the imposed boundary conditions, we write the last expression in the following form:

$$V_{x_0}(z) = \frac{\beta_1^{k_t}(z)\beta_2^{x_0}(z) - \beta_1^{x_0}(z)\beta_2^{k_t}(z)}{\beta_1^{k_t}(z) - \beta_2^{k_t}(z)}. \quad (10)$$

Taking into account that $\beta_1(z)\beta_2(z) = P_- P_+^{-1}$, expression (10) can be written as follows:

$$V_{x_0}(z) = \left(\frac{P_-}{P_+}\right)^{x_0} \frac{\beta_1^{k_t - x_0}(z) - \beta_2^{k_t - x_0}(z)}{\beta_1^{k_t}(z) - \beta_2^{k_t}(z)}. \quad (11)$$

The generating function $V_{x_0}(z)$ of the probability $V_{x_{0,n}}$ of the error exceeding the lower boundary of the tolerance field can now be determined without solving the difference equation (6) by replacing in expression (11) $P_+$, $P_-$, $x_0$ by $P_+$, $P_-$, $k_t - x_0$:

$$V_{x_0}(z) = \left(\frac{P_-}{P_+}\right)^{k_t - x_0} \frac{\beta_1^{x_0}(z) - \beta_2^{x_0}(z)}{\beta_1^{k_t}(z) - \beta_2^{k_t}(z)}. \quad (12)$$

Now, in order to find expressions for the conditional probabilities $U_{x_{0},h}$ and $V_{x_{0},h}$, which determine the probabilities of metrological failures, $P^+$, $P^-$ and the value of the confidence indicator $D$, it is necessary to determine the coefficients at $z^h$ in the expansion of the corresponding generating functions.

In order to simplify the subsequent mathematical transformations, we take the initial value of the SI error equal to half of the tolerance field, etc.$x_0 = \frac{k_t}{2}$, and finally we obtain the following expression for the confidence indicator:

$$D = 1 - \frac{2}{k_t}\sqrt{P_+ P_-}\left[\left(\frac{P_+}{P_-}\right)^{\frac{k_t}{4}} + \left(\frac{P_-}{P_+}\right)^{\frac{k_t}{4}}\right]\sum_{i=1}^{k_t-1}\frac{1-(2\sqrt{P_+P_-}cos\frac{\pi i}{k_t})^{N_t}}{1-2\sqrt{P_+P_-}cos\frac{\pi i}{k_t}} sin\frac{\pi i}{k_t} sin\frac{\pi i}{2}, \quad (13)$$

where $i = 0, 1, 2, 3\ldots$

Expression (13) is a mathematical relationship in which the confidence indicator D will depend on four variables:

$$D = f(P_+, P_-, k_t, N_t).$$

To analyze the obtained estimate, let us construct graphs of the reliability indicator D from the corresponding arguments $P_+, P_-, k_t, N_t$. To do this, we transform expression (13) to the following form:

$$D = 1 - Z \cdot \sum_{i=1}^{k_t-1}\frac{1-L^{N_t}}{1-L} \cdot F, \quad (14)$$

where $Z = \frac{2}{k_t}\sqrt{P_+P_-} \cdot \left|\left(\frac{P_+}{P_-}\right)^{\frac{k_t}{4}} + \left(\frac{P_-}{P_+}\right)^{\frac{k_t}{4}}\right|$;

$$L = 2\sqrt{P_+P_-} \cdot cos\frac{\pi i}{k_t};$$

$$F = sin\frac{\pi i}{k_t} \cdot sin\frac{\pi i}{2}.$$

The table contains calculations of the reliability indicator D depending on the interval of storage of the accumulated error within the tolerance field $\pm\Delta_{ad}$ at the values of $k_t$ equal to 20, 10, 5, 4, 3, respectively, and the probabilities of increment of the "positive" and "negative" errors equal to 0,5.

**Values of the confidence indicator *D* from the interval $N_t$**

| $N_t$ | $D_1(k_t=20)$ | $D_2(k_t=10)$ | $D_3(k_t=5)$ | $D_4(k_t=4)$ | $D_2(k_t=3)$ |
|---|---|---|---|---|---|
| 0,0 | 1,00 | 1,00 | 1,00 | 1,00 | 1,00 |
| 10,0 | 1,00 | 0,78 | 0,21 | 0,03 | 0,00 |
| 20,0 | 0,95 | 0,47 | 0,08 | 0,00 | 0,00 |
| 30,0 | 0,86 | 0,29 | 0,06 | 0,00 | 0,00 |
| 40,0 | 0,77 | 0,17 | 0,06 | 0,00 | 0,00 |
| 50,0 | 0,68 | 0,11 | 0,06 | 0,00 | 0,00 |
| 60,0 | 0,60 | 0,06 | 0,06 | 0,00 | 0,00 |
| 70,0 | 0,53 | 0,04 | 0,06 | 0,00 | 0,00 |
| 80,0 | 0,47 | 0,02 | 0,06 | 0,00 | 0,00 |
| 90,0 | 0,41 | 0,01 | 0,06 | 0,00 | 0,00 |

Based on the calculations obtained, the graphs presented in Fig. 2. It can be seen from the graphs that the value of the reliability indicator at fixed values of $P_+$ and $P\_$ decreases with an increase in the required interval of error evolution ($N_t$) and a decrease in the value of the tolerance field $k_t$. At the same time, for fixed values of $k_t$ and $N_t$, the value of the reliability indicator depends on the ratio of the probabilities $P_+$ and $P\_$, taking the maximum value at their equal values, i.e. when $P_+ = P\_ = 0.5$. In this case, the process of evolution of the error has the character of a symmetric random process and falls at $P_+ \neq P\_$, i.e., in this case, a predominant direction appears in the process of evolution of the error.

The assessment of the metrological reliability of the measuring instrument using the generating functions was carried out for the model of gradual failures with a discrete change in the error over time. This assessment of metrological reliability can be generalized for measuring instruments with continuous variation of the error over time. In this case, expression (4) takes the form

$$D = 1 - \int_0^{\tau_*} [U(x_0 t) + V(x_0 t)] dt, \qquad (15)$$

where$U(x_0 t), V(x_0 t)$ - are asymptotic approximations to the probabilities $U_{x_0,h}$ and $V_{x_0,h}$.

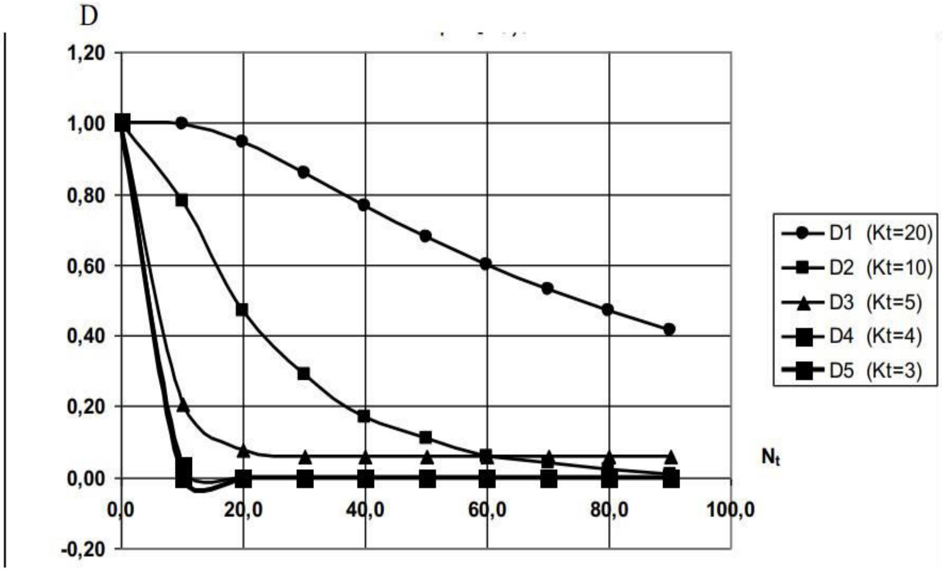


Fig. 2. Graphs of dependencies $D = f\,(N_t)$ at $P_+ = P_- = 0.5$

However, the issue of obtaining a generalized assessment of metrological reliability for SI with a continuous change in error over time is beyond the scope of this article. The obtained value of the reliability indicator with the use of generating functions can be taken as an assessment of the metrological reliability of the SI and, on its basis, implement an effective algorithm for assessing the metrological reliability of the SI by means of information technology. This approach is relevant, as noted earlier, for SI, in which the model of gradual failures with a discrete change in the error in time, which is characteristic of a certain class of SI, for example, for the class of technical means of timekeeping, is presented as a model of the evolution of the error. This determines the restrictions on its use.

## CONCLUSIONS

1. Issues related to the assessment of the SI reliability are systematized. Reliability is assessed according to various indicators, while the most important indicators for SI are metrological reliability indicators.

2. For measuring instruments having a model of gradual failures with a discrete change in error in time, a reliability indicator is proposed as the main indicator of the metrological reliability of the measuring instrument, which allows predicting the metrological failures of the measuring instrument with a given probability.

3. For the analytical determination of the reliability indicator, the method of generating functions is used, which makes it possible to significantly simplify its finding, which will further create conditions for automating the process of finding the reliability indicator using information technologies, in which an effective algorithm based on generating functions can be implemented.

4. The results obtained in the article can be extended to other types of SI.

**REFERENCES**


1. Metrology, standardization and certification: textbook for students. Higher education institutions on spec. eg. prep. "Transport machines and transport and technological complexes" and "Operation of ground transport and transport equipment"; additional UMO / A.I. Aristov [et al.]. Moscow: Infra-M, 2014. 256 p. (Higher education) (Bachelor's degree).
2. GOST 8.009-84. Normalized metrological characteristics of measuring instruments.
3. MasharipovSh.M., Mamatqulov M.N., Erkaboyev A.X. Metrological Accuracy and Estimation of Extended Uncertainty of Pressure Gauge in Real Conditions of Explaution. International Journal of advanced research of science, Engineering and Technology. Vol. 7, Issue 5, May 2020. Pp 13801-13805.
4. Metrology, standardization and certification: textbook for universities / A.G. Sergeev, V.V. Therehere. Moscow: Yurait, 2011. 820 p. (Fundamentals of science).
5. Ibrokhimov, J. M. (2021). Features of methods of optimising calculation of parameters the combined solar power installations. ACADEMICIA: An International Multidisciplinary Research Journal, 11(5), 1043-1047.
6. Ibrokhimov, J. M. (2020). Application of the solar combined systems consisting of the field of flat and parabolocylindrical collecting channels for hot water supply of the industrial factories. ACADEMICIA: AN INTERNATIONAL MULTIDISCIPLINARY RESEARCH JOURNAL, 10(12), 1293-1296.