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SHUR INEQUALITY AND ITS AMAZING APPLICATIONS

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ABSTRACT

The subject of inequalities is one of the important areas of mathematics. So far, there have been many confirmations and conclusions on inequalities. This article will cover one of the important types of inequalities Shur inequality. This inequality has many wide applications, taking an important place in mathematical olympiads. In the case when we enter the level rows, we get some important results and study their application in matters. At the same time, we cite the methods of solving the issues proposed in the prestigious international Olympiads in mathematics by applying the inequality of Shur.

KEYWORDS: *Shur Inequality, Level Rows, Gamma Function Of Eyler.*

INTRODUCTION

The fact that Shur inequality is recommended in a number of prestigious Olympiads with its wide application and conducted in mathematics, issues that can be solved in exactly the same way, necessitates our deeper study of inequality. In this article we will look at the cases of the application of Shur inequality and some of its manifestations. In the case when we enter the level rows, we get some important results and study their application in matters. At the same time, we cite the methods of solving the issues proposed in the prestigious international Olympiads in mathematics by applying the inequality of Shur. In the modern era, which is developing rapidly now, a new look at all aspects of science, technology and industry, non-standard thinking is extremely important, and the development of such a worldview, thinking takes some time and labor. We also have directions that require a lot of non-standard thinking in the field of mathematics, and now the development of such views is the main task. So in this article, we will cite ways to solve many other, sufficiently high-complexity issues through a single inequality. First we will consider the confirmation that Shur brought inequality and prove this famous inequality.

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Theorem 1 (Shur inequality) Let's say x, y and z s be positive real numbers. Then it is desirable $\alpha > 0$ for the following inequality fitting.

$$\sum_{cyc} x^{\alpha}(x-y)(x-z) \geq 0$$

Proof: it can be said that since the inequality seen is symmetrical in relation to x, y ,z , without prejudice to the generality $x \geq y \geq z$. Now we write the left side of the inequality that is required to be proved come to the following view.

$$\sum_{cyc} x^{\alpha}(x-y)(x-z) = x^{\alpha}(x-y)(x-z) + y^{\alpha}(y-x)(y-z) + z^{\alpha}(z-x)(z-y)$$

$$= (x-y)\big(x^{\alpha}\,(x-z) - y^{\alpha}(y-z)\big) + z^{\alpha}\,(z-x)(z-y)$$

Apparently each part of the sum of the obvious is positive.

Equal $x = y = z$ or x , y , z two of the tokens are mutually equal and are executed when the third one is equal to zero. Proven.

In matters it is mainly effective to initially perform the appropriate replacement, and then apply Shur inequality.

Issue 1 (IMO 2000) For real positive numbers a, b, c, abc=1 if the condition is reasonable, prove the following inequality

$$\left(a-1+\frac{1}{b}\right)\left(b-1+\frac{1}{c}\right)\left(c-1+\frac{1}{a}\right) \leq 1$$

Proof: as is known, the inequality is strong equal to the same-sex inequality following$(a - (abc)^{\frac{1}{3}} + \frac{(abc)^{\frac{2}{3}}}{b})\ (b - (abc)^{\frac{1}{3}} + \frac{(abc)^{\frac{2}{3}}}{c})(c - (abc)^{\frac{1}{3}} + \frac{(abc)^{\frac{2}{3}}}{a} \leq abc$

If $a = x^3$,$b = y^3$,$c = z^3$ $x, y, z > 0$ if we apply substitution, then this inequality comes in the form of the following.

$$(x^3 - (xyz) \quad + \frac{(xyz)^2}{y^3})(y^3 - (xyz) \quad + \frac{(xyz)^2}{z^3})\ (z^3 - (xyz) \quad + \frac{(xyz)^2}{x^3}) \leq x^3y^3z^3$$

Or, in short, we can bring the following look

$$3(xyz)^3 + \sum_{cyc} x^6y^3 \geq \sum_{cyc} x^4y^4z + \sum_{cyc} x^5y^2\,z^2 + \sum_{sym} x^3 \ \geq 2\sum_{sym} x^2y$$

Now, having simplified the last inequality, we can come up with the following.

$$3(x^2y)(y^2z)(z^2x) + \sum_{cyc}(x^2y)^3 \ \geq \sum_{sym}(x^2y)^2(y^2z)$$

And our last inequality proved inequality, considering that the inequality of Shur is one of the private cases.

Issue 2 (USA TST 2003) If $,b,c \in (0,\frac{\pi}{2})$, then prove the following inequality $\frac{\sin(a)\sin(a-b)\sin(a-c)}{\sin(b+c)} + \frac{\sin(b)\sin(b-c)\sin(b-a)}{\sin(a+c)} + \frac{\sin(c)\sin(c-a)\sin(c-b)}{\sin(a+b)} \geq 0$

Proof : First we do the following replacement

$x = \sin a\ , y = \sin b\ , z = \sin c$ get $a, b, c \in (0, \frac{\pi}{2})$ so

$x, y, z > 0$ it turns out that. It is not difficult to make sure that the following attitude is appropriate

$\sin(a)\sin(a-b)\sin(a-c)\sin(a+b)\sin(a+c) = x^2(x^2-y^2)(x^2-z^2)$

For other hads, we write the same relationship and bring the inequality that is required to be proved in the following form.

$$\sum_{cyc} x(x^2-y^2)(x^2-z^2) \geq 0$$

So inequality $x = \sqrt{u}$, $y = \sqrt{v}$, $z = \sqrt{w}$ if we do the replacements, it will come in the form of the following.

$$\sum_{cyc} \sqrt{u}\,(u-v)(u-w) \geq 0$$

This is the inequality of Shur $\alpha = \frac{1}{2} > 0$ private case in case.

Now let's look at the proof of some inequalities with the inclusion of level rows.

Theorem 2 If $f(x) = \sum_{k=0}^{\infty} a_k x^k$, $a_k \geq 0, k \in N$,

Then prove the following inequality

$$(x-y)(x-z)f(x) + (y-z)(y-x)f(y) + (z-x)(z-y)f(z) \geq 0$$

There $\forall\, x, y, z \geq 0$

Proof: in the case where Shur applies inequality, we can write the following reasonable inequality.

$$(x-y)(x-z)a_k x^k + (y-z)(y-x)a_k y^k + (z-x)(z-y)a_k z^k \geq 0$$

And proceeding from this, we can proceed to level rows.

$$(x-y)(x-z)\sum_{k=0}^{\infty} a_k x^k + (y-z)(y-x)\sum_{k=0}^{\infty} a_k y^k + (z-x)(z-y)\sum_{k=0}^{\infty} a_k z^k \geq 0$$

This while proof itself of the required inequality. Proven proof, using

Theorem 2, we bring the inequality that gives an excellent effect.

Issue 3 : If $\alpha \in R$ va $x, y, z \geq 0$, then prove the following inequality

$$(x-y)(x-z)ch(\alpha lnx) + (y-z)(y-x)ch(\alpha lny) + (z-x)(z-y)ch(\alpha lnz) \geq 0$$

There ch(t) hyperbolic cosine function, $ch(t) = \frac{e^t + e^{-t}}{2}$

Proof: using Theorem 2, we can write the following.

$$\sum_{cyc}(x-y)(x-z)x^{\alpha} \geq 0 \qquad va \quad \sum_{cyc}(x-y)(x-z)x^{-\alpha} \geq 0$$

By adding two inequalities, we get the following inequality.

$$0 \leq \sum_{cyc}(x-y)(x-z)x^{\alpha} + \sum_{cyc}(x-y)(x-z)x^{-\alpha} = \sum_{cyc}(x-y)(x-z)(x^{\alpha} + x^{-\alpha})$$

So $x^{\alpha} + x^{-\alpha} = e^{\alpha lnx} + e^{-\alpha lnx} = 2ch(\alpha lnx)$ if we take into account that this is the case, we will come to the following.

$$\sum_{cyc}(x-y)(x-z)(x^{\alpha} + x^{-\alpha}) = 2\sum_{cyc}(x-y)(x-z)ch(\alpha lnx) \geq 0$$

Proven.

Now we will come up with some issues that are intended to work independently.

Issue 1: prove that the following inequality is reasonable for the real numbers of a, b, c positive optional

$$27 + (2 + \frac{a^2}{bc})(2 + \frac{b^2}{ac})(2 + \frac{c^2}{ab}) \geq 6(a+b+c)(\frac{1}{a} + \frac{1}{b} + \frac{1}{c})$$

Issue 2: If $a, b, c, d > 0$ va $a + b + c + d = 1$, prove the following inequality

$$a^3 + b^3 + c^3 + abcd \geq min\left\{\frac{1}{4}, \frac{1}{9} + \frac{d}{27}\right\}$$

KVANT 1993

REFERENCES:

1. Hojoo Lee. Topics in Inequalities-Theorems and Techniques. Seoul: 2004.
2. Andreescu T., Dospinescu G., Cirtoaje V., Lascu M. Old and new inequalities. Gil Publishing House, 2004.
3. Mathematical Olympiads, Problems and solutions from around the world, 1998-1999. Edited by Andreescu T. and Feng Z. Washington. 2000.
4. Math Links, http://www.mathlinks.ro
5. Art of Problem Solving, http://www.artofproblemsolving.com
6. Math Pro Press, http://www.mathpropress.com
7. D.S. Mitrinovic, Analytic inequalities, Springer Verlag, 1970.

8. K.S.Kedlaya, A<B, http://www.unl.edu/amc/a-activities/a4-for-students/s-index.html

9. T.J.Mildorf, Olympiad Inequalities, http://web.mit.edu/tmildorf

10. Алфутова Н.Б., Устинов А.В. Алгебра и теория чисел. Сборник задач для математических школ. М.: МЦНМО, 2002.

11. A.Engel. Problem-Solving Strategies. Parts 1, 2. Springer-Verlag New York Inc. 1998.

12. Ayupov Sh., Rihsiyev B., Quchqorov O. «Matematika olimpiadalar masalalari» 1,2 qismlar. T.: Fan, 2004.