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OWN TORSIONAL VIBRATIONS OF A CYLINDRICAL SHELL IN AN ELASTIC MEDIUM

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ABSTRACT

The article covers that we consider the own axisymmetric torsional vibrations of an infinitely long cylindrical shell in an elastic inertial medium. The dependences of the first shell oscillation frequency in an elastic medium on the dimensionless wavelength are obtained.

KEYWORDS: *Natural Torsional Vibrations, Axisymmetric Vibrations, Cylindrical Shell, Vibration Frequency.*

INTRODUCTION

Axisymmetric natural torsional vibrations of an infinitely long cylindrical shell in an elastic inertial medium are investigated. The movement of the shell and the massif is described by the dynamic equations of the theory of elasticity.

The equation of motion of the Kirchhoff - Love shell, taking into account the response of the array, can be written in the form.

$$\frac{\partial^2\vartheta}{\partial x^2} - \frac{P}{G}\frac{\partial^2\vartheta}{\partial t^2} = \frac{1}{Gh}q_c \qquad (1)$$

Taking into account that only shear waves are excited for a torsional load, we obtain the equation of motion of the medium.

$$\frac{\partial^2 u_0}{\partial r^2} - \frac{l}{r}\frac{\partial u_0}{\partial r} - \frac{u_0}{r^2} + \frac{\partial^2 u_0}{\partial x^2} = \frac{P_c}{G_c}\frac{\partial u_0}{\partial t^2} \qquad (2)$$

The values of nonzero components of the stress tensor in the medium are determined through tangential displacements by the formulas

$$\sigma_{r0} = G_c\left(\frac{\partial u_0}{\partial r} - \frac{u_0}{r}\right); \;\; \sigma_{x0} = G_c\frac{\partial u_0}{\partial x} \qquad (3)$$

The boundary conditions of the problem at r = R have the form

$$u_0 = v, \sigma_{r0} = -q_c; \quad (4)$$

Considering the axisymmetric vibrations of the shell, we find solutions to equations (1) and (2):

$$(\vartheta, u_0, q_c) = \{\vartheta_0, U(r), q_{c0}\} cosmxe^{-i\omega t} \qquad (5)$$

where π / m is the wavelength along the generatrix; ω is the circular frequency of natural vibrations.

Substituting expression (5) into (1), we obtain the relationship between the amplitudes of the array reaction, and the displacements of the shell

$$q_{c0} = Gx^2(x^2\omega^2 - \delta^2)U_0; \qquad x = \frac{h}{R}, \quad \delta = mR;$$

$$\omega^2 = \frac{\rho R^2}{Gx^2}\omega^2; \qquad U_0 = U_0/h; \qquad (6)$$

Dependences (6) allow the boundary condition for the medium at r = R to be written in the form

$$\sigma_{r0}^0 = -Gx^2(x^2\omega^2 - \delta^2)U_0 \qquad (7)$$

Substituting expression (5) into (2), we obtain

$$\frac{\mathrm{d}^2\mathrm{U}_0}{\mathrm{d}\vec{\mathrm{r}}^2} + \frac{1}{\vec{\mathrm{r}}}\frac{\mathrm{dU}_0}{\mathrm{d}\vec{\mathrm{r}}} - \frac{\mathrm{U}_0}{\vec{\mathrm{r}}^2} - (\delta^2 - \alpha^2)\mathrm{U}_0 = 0 \qquad (8)$$

where $\vec{r} = \frac{r}{R}$; $U_0 = \frac{U}{h}$; $\alpha^2 = \frac{x^2\gamma}{\rho\omega^2}$; $\gamma = \frac{G}{G_c}$; $\rho = \rho/\rho_c$

Solving equations (8) taking into account the condition of damping of oscillations at infinity, we have

$$U_0 = AK_1(\beta r *); \alpha < \delta; U_0 = A/r; \alpha = \delta; \qquad (9a)$$

$$U_0 = AH_1^{(2)}(\bar{\beta}r); \; \alpha > \delta; \qquad (9b)$$

Here $\beta = \sqrt{\delta^2 - \alpha^2}; \bar{\beta} = \sqrt{\alpha^2 - \delta^2}$

$K_1(x)$; $H_1^{(2)}(x)$- Macdonald and Hankel functions.

If we do not take into account the inertia of the medium, then $\rho_c = 0$ $(\alpha = 0)$and the solution to equation (8) will be the equality

$$U_0 = AK_1(\delta r *) \qquad (10)$$

Substituting solutions (9) into (3) taking into account the boundary condition (7), we obtain the characteristic equation:

$$x\gamma(\delta^2 - x^2\omega^2) + 2 - \bar{\beta}\frac{H_0{}^{(2)}(\bar{\beta})}{H_1{}^{(2)}(\bar{\beta})} = 0,\ \ \alpha > \delta \qquad (11)$$

For $\alpha < \delta$ in equation (11), instead of the last term, it is necessary to insert with a plus sign the expression $\frac{\beta K_0(\beta)}{K_1(\beta)}$

The condition $\alpha = \delta$ is satisfied for $\rho *= \gamma + 2/x\delta^2$.

This case is of no practical interest, since for most materials $\rho *< \gamma$.

Note that the vibration frequency of the shell in vacuum $(\gamma = \infty)$

$$\omega *= \delta/x \qquad (12)$$

In a non-inertial environment

$$\omega^2 = (x\gamma\delta^2 + 2 + \delta K_0(\delta)/K_1(\delta))/x^3\gamma \qquad (13)$$

For a shell in an inertial medium, the natural vibration frequencies are determined by solving the transcendental equation (11) on a computer.

2. Let us describe the vibrations of the shell by equations of the Timoshenko type. Then for axisymmetric torsional motion we have:

$$\frac{\partial^2\vartheta}{\partial x^2} - \frac{K^2}{R}\beta - \frac{P}{G}\frac{\partial^2\vartheta}{\partial t^2} = \frac{1}{Gh}q_c$$

$$\frac{\partial^2\beta}{\partial x^2} - 12\frac{k^2}{h^2}\beta - \frac{P}{G}\frac{\partial^2\beta}{\partial t^2} = \frac{6}{Gh^2}q_c \qquad (14)$$

Here $\beta$is the angle of rotation of the normal in the tangential direction, k2 is the Timoshenko coefficient.

The boundary conditions for the medium at r = R are as follows:

$$u_0 = r + \frac{h}{2}\beta; \sigma_{r0} = -q_c \qquad (15)$$

Representing the solution of equations (14) in the form (5), we determine the parameters q_c , $\alpha_0$through the displacements v_0

$$q_{c0} = Gx^3\frac{\alpha_1\alpha_2}{Gk^2 + x\alpha_1}\overline{\vartheta_0}; \alpha_0 = G\gamma\frac{\alpha_2}{GR^2 + x\alpha_1}\overline{\vartheta_0}$$

$$\alpha_1 = x^2\omega^2 - \delta^2 - 12\frac{k^2}{x^2}; \alpha_1 = x^2\omega^2 - \delta^2 \ (16)$$

Taking into account expression (16), we transform the boundary condition for the medium.

For r=1 $\sigma^2{}_{r0} = -Gx^2\frac{a_2}{a_3a_4}U_0$

where $a_3 = 1 + \frac{k^2}{x}\frac{6}{a_1}$, $a_4 = 1 + \left(\frac{3a_2}{a_1a_3}\right)$ (17)

Since the solution of the equation of motion of the medium does not depend on the adopted shell model, the further derivation of the equations is similar to that considered above for the Kirchhoff-Love shell. The choice of the shell model affects only the form of the boundary conditions (17).

Instead of the characteristic equation (11), we write equations of the form:

$$xy^{\frac{\alpha_2}{\alpha_3\alpha_4}} + 2 - \bar{\beta}\frac{H_0{}^{(2)}(\bar{\beta})}{H_1{}^{(2)}(\bar{\beta})} = 0\alpha > \delta \qquad (18)$$

For the case $\rho_c = 0$ $(\alpha = 0)$, we obtain the characteristic equation for the Timoshenko shell in an inertial-free medium. At $G_c = 0$, we find the frequencies of torsional vibrations of the Timoshenko shell in vacuum

$$\omega *= \delta/x; \omega_2 *= \frac{1}{x}\sqrt{\delta^2 + 12\frac{k^2}{x^2}} \qquad (19)$$

The first frequency, as for the Kirchhoff-Love shell, corresponds to the rotation of the section like a ring, the second to the form of vibrations caused by the rotation of the normal in the tangential direction.

The first frequency, as for the Kirchhoff-Love shell, corresponds to the rotation of the section like a ring, the second to the form of vibrations caused by the rotation of the normal in the tangential direction.

Putting in equations (11), (18) $h = 0$ $(\alpha = 0)$ and introducing the dimensionless frequency $\omega^2 = \frac{\rho_c R^2 \omega^2}{G_c}$, we obtain the characteristic equation for the natural axisymmetric torsional vibrations of an elastic inertial mass with a cylindrical cavity.

$$\overline{\beta_2} H_0{}^{(2)}(\overline{\beta_2}) - 2H_1{}^{(2)}(\overline{\beta_2}) = 0 \qquad (20)$$

Moreover, $\omega > \delta, \overline{\beta_2} = \sqrt{\omega^2 - \delta^2}$

3. Let us obtain the exact solution of the problem for the case when the motion of the shell is described by equation (2). Then the boundary conditions will be as follows:

For $r = Ru_\theta{}^{(1)} = u_\theta; \sigma_{r0}{}^{(1)} = \sigma_{r0}$

for $r = R - h\sigma_{r0}{}^{(1)} = 0;$ (21)

Here, index 1 denotes a shell.

We write the general solution of the equations of motion of a cylindrical layer in the form

$$U_0{}^{(1)} = A_1 K_1(\beta_1 r) + \beta_1 I_1(\beta_1 r); x\omega < \delta;$$

$$U_0{}^{(1)} = A_1 \frac{1}{r} + \beta_1 r \qquad x\omega = \delta;$$

$$U_0{}^{(1)} = A_1 Y_1(\overline{\beta_2} r) + \beta_1 J_1(\overline{\beta_2} r); \quad x\omega > \delta; \qquad (22)$$

Where$\beta_1 = \sqrt{\delta^2 ->^2 \omega^2}\overline{\beta_1} = \sqrt{>^2 \omega^2 - \delta^2}$

J1 (x), Y1 (x) are the Besell functions of the first and second kind.

Substituting expression (22), (9) into (3) and satisfying boundary conditions (21), we obtain the characteristic equation

$$det\|a_{ij}\| = 0, \quad i,j = 1,2,3 \tag{23}$$

Determinant elements:

For $< \sqrt{\frac{\rho}{\gamma}}\delta$, $\sqrt{\frac{\rho}{\gamma}}\delta < x\omega < \delta, \quad \delta < x\omega$has a different form.

For example, for the third case:

$$a_{11} = Y_1(\overline{\beta_1}); a_{12} = J_1(\overline{\beta_1}) a_{13} = -H_1{}^{(2)}(\overline{\beta_1})$$

$$a_{21} = \overline{\beta_1} Y_0(\overline{\beta_1}) - 2Y_1(\overline{\beta_1}); \quad a_{22} = \overline{\beta_1} J_0(\overline{\beta_1}) - 2J_1(\overline{\beta_1})$$

$$a_{23} = -\left(\frac{1}{Y}\right)\left(\bar{\beta} H_0{}^{(2)}(\bar{\beta})\right) - 2H_1{}^{(2)}(\bar{\beta}); \; a_{31} = \overline{\beta_1} Y_0(\overline{\beta_1}\varepsilon) - \left(\frac{2}{\varepsilon}\right) Y_1(\overline{\beta_1}\varepsilon);$$

$$a_{32} = \overline{\beta_1} J_0(\overline{\beta_1}\varepsilon) - \left(\frac{2}{\varepsilon}\right) J_1(\overline{\beta_1}\varepsilon); a_{33} = 0; \varepsilon = 1 - x;$$

4. For all considered types of the equation of motion of the shell, numerical results are obtained. Figure 1 shows the dependence of the dimensionless frequency of torsional vibrations on the relative thickness of the shell. Calculation 1 is carried out for δ=3, γ=30, ρ=4, k^2=2/3.

Curve 1 corresponds to the first mode of movement, 2-to the second. 3-rd third. The solid line marks the shell in the inertial mass, the dashed line - in the inertial-free medium, and the dotted line - in the vacuum.

Calculations have shown that for the first mode of motion, the results for all three theories practically coincide, and the effect of the medium on the vibration frequency is especially significant for relatively thin shells (x <0.03). If the inertia of the medium is not taken into account, the value of the first frequency is overestimated. The third frequency (the second for the Timoshenko shell) is almost unaffected by the medium. Thus, for a shell with a thickness of x <0.07, the obtained frequency values practically coincide with the exact ones.

If the shell motions are described by the equations of the theory of elasticity, a second mode appears (curve 2), associated with the uneven twisting of the cylinder along the thickness, caused by the presence of an elastic mass.

Thus, to determine the first vibration frequency of the shell in an elastic medium, it is necessary to use equations (11), (formula (13) gives overestimated values), for higher vibration frequencies - equation (23).

The dependence of the first vibration frequency of the shell in an elastic medium on the dimensionless wavelength for different relative rigidity of the array is shown in Fig. 2. (x = 0.01). For curves

$$1 - \gamma = 120, \quad \rho = 4; 2 - \gamma = 30, \quad \rho = 4; 3 - \gamma = 5, \quad \rho = 1$$

The solid line shows the oscillation frequencies of the shell in the inertial array, the dashed line - inertia-free, the dash-dotted line - in the vacuum. As seen from Fig. 2. with an increase in the rigidity of the array, the error increases, which is introduced without taking into account the inertia of the medium (especially in the region of short waves). Using asymptotic representations of cylindrical functions, we obtain formulas for the first frequency of natural oscillations in the case of long waves $\omega^2 = \frac{\bar{\rho}}{x^2\gamma}\left(a + \sqrt{a^2 + b}\right)$

There$a = \frac{64x\gamma\delta^2+96-9x\bar{\rho}}{128x\overline{\rho_0}}$; $b = \frac{9(2+x\gamma\delta^2)}{64x\overline{\rho_0}}$

If we do not take into account the inertia of the array $\bar{\omega}_0{}^2 = \frac{(x\gamma+1)\delta^2+2}{x^3\gamma}$

Then on 1.5δ≤1, the results by asymptotic formulas practically coincide with the exact solution.

In conclusion**,** to determine the first vibration frequency of the shell in an elastic medium, it is necessary to use the equation of motion of the Kirchhoff - Love shell; for higher vibration frequencies, the equation of motion of the theory of elasticity.

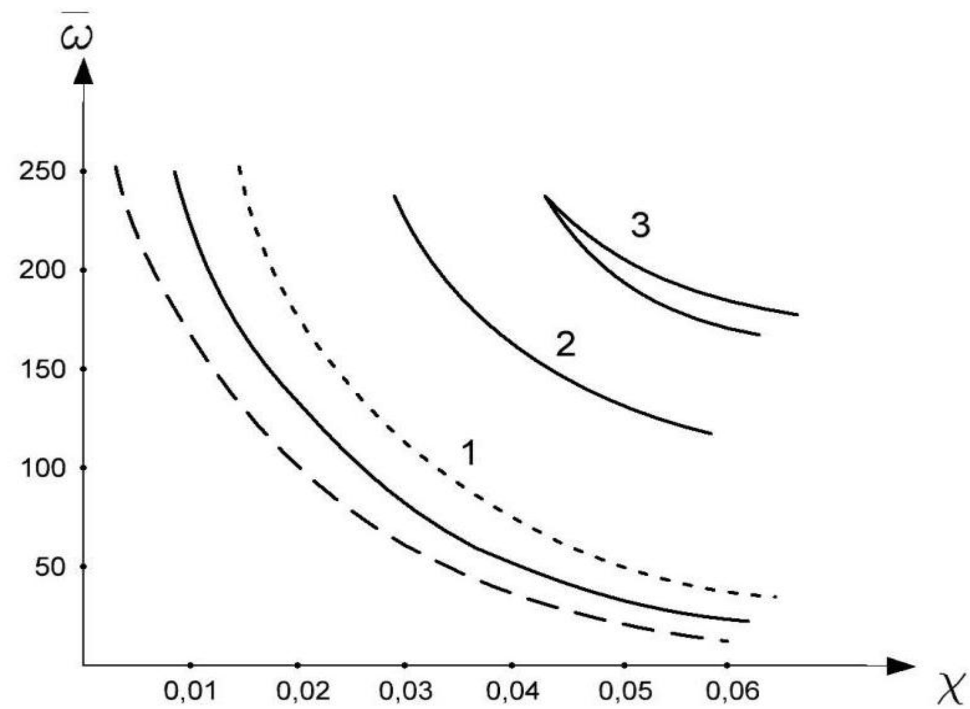

Fig.1. Changes in natural frequencies depending on x.

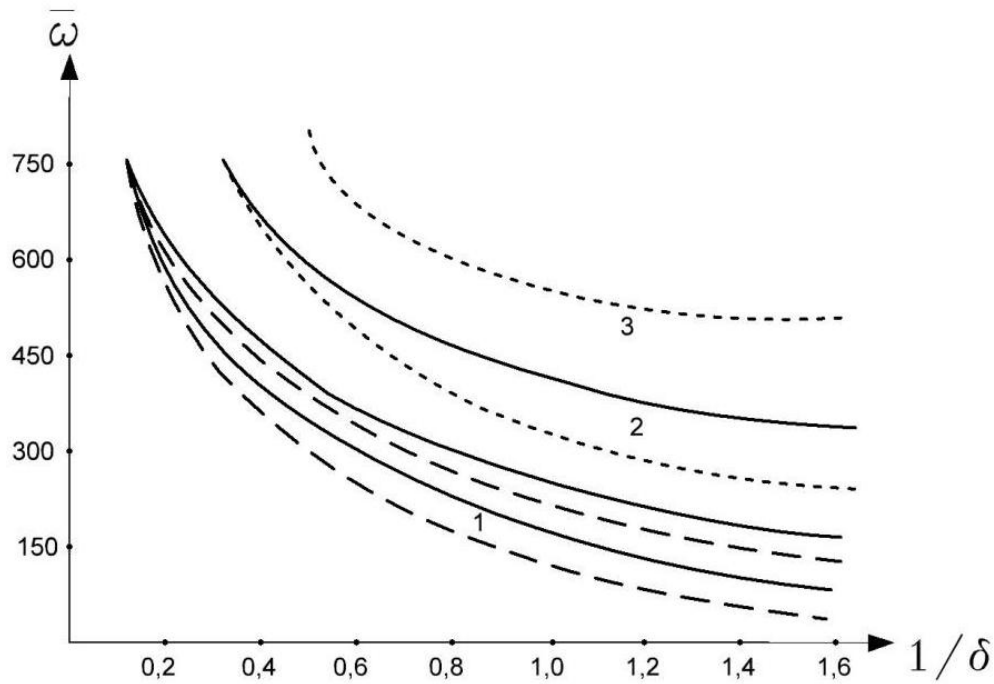

Fig.2. Change of natural frequencies from 1 / δ.

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