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ON MODELING OF MECHANICAL VIBRATIONS OF ORTHOTROPIC BOARDS IN ELECTRONIC DEVICES

Abjalilov Sanakul Xujamovich*; Ashurova Dilfuza Nabiyevna; Begmurodov Otabek Ahmadovich*****

*Associate Professor,
Candidate of physical and Mathematical Sciences,
Navoi State Pedagogical Institute,
UZBEKISTAN

**Associate Professor,
Doctor of Philosophy in Pedagogical Sciences,
Navoi State Pedagogical Institute,
UZBEKISTAN

***Teacher, NSPI
UZBEKISTAN

ABSTRACT

Decisions of the homogeneous and non-uniform equation of a bend orthotropic on mechanical properties of the rectangular plate, received are described by a recurrently-operational method. For an illustration of algorithm of the decision of problems the recurrently-operational method results numerical results of test problems for the non-uniform equation, satisfying to initial and boundary conditions.

KEYWORDS: *Electronic Equipment, Orthotropic Plate, Lame Equation, Recurrent Operator Method.*

INTRODUCTION

In modern conditions, electronic equipment, in addition to meeting the main set of technical characteristics, is subject to a number of strict requirements aimed at improving the design of electronic equipment that is resistant to mechanical and thermal disturbances [1].

In the monograph [2], discrete models of devices are considered without taking into account the anisotropy of electronic boards. However, when placing individual electronic devices on the boards, some structural anisotropy of mechanical and thermal properties is created. Therefore, when placing individual elements in an electronic circuit, it is desirable to ensure the regularity of the structure so that the board can be considered as an orthotropic plate.

This article deals with the problem of mechanical vibrations of an orthotropic plate.

As is known, the problem of the eigenvalues of an orthotropic plate is reduced to solving the differential equation of plate deflections [3, 4, and 5]

$$L(w) \equiv \left(b_0 \frac{\partial^4}{\partial x^4} + b_1 \frac{\partial^4}{\partial x^2 \partial y^2} + b_2 \frac{\partial^4}{\partial y^4} + \frac{\partial^2}{\partial t^2}\right) w = 0, \qquad (1)$$

Satisfying the initial and boundary conditions. Here

$$b_0 = \frac{1}{\rho} D_1;\ \ b_1 = \frac{2}{\rho} D_3;\ \ b_2 = \frac{1}{\rho} D_2;\ \ f = \frac{1}{\rho} f_q;\ \ \rho = \frac{h_0 \gamma}{g};$$

$$D_1 = \frac{E_1 h_0^3}{12(1-\nu_1\nu_2)};\ \ D_2 = \frac{E_2 h_0^3}{12(1-\nu_1\nu_2)};\ \ D_3 = D_1\nu_2 + 2D_k;\ \ D_k = \frac{G h_0^3}{12},$$

h_0 −Plate thickness; γ −specific gravity of the plate material; E_1, E_2, ν_1, ν_2, G −Young's modules, Poisson's coefficients, and the shear modulus for the principal directions; D_1, D_1 - cylindrical stiffness; λ, μ −Lame constants, $w(x, y, t)$ −deflections of the median surface of the plate.

Problem statement

Find deflections $w(x, y, t)$ the median surface of an orthotropic articulated plate satisfying equation (1) and the following boundary and initial conditions:

$$w\Big|_{\substack{x=0,a\\ y=0,b}} = \frac{\partial^2 w}{\partial x^2}\Big|_{x=0,a} = \frac{\partial^2 w}{\partial y^2}\Big|_{y=0,b} = 0 \qquad (2)$$

$$w\Big|_{t=0} = 0,\ \ \frac{\partial}{\partial t} w\Big|_{t=0} = f(x, y). \qquad (3)$$

To solve the problem (1) - (3), we apply the recurrent operator method [6,7].

Following the recurrent-operator method, the solution of the homogeneous equation (1) is sought in the form

$$w_r[g_r(x, y)] = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} Q_{i,j} \frac{\partial^{2i+2j}}{\partial x^{2i} \partial y^{2j}} g_r(x, y) \frac{t^{i+j+r}}{(i+j+r)!}, \qquad r = 0,\ 1. \ (4)$$

Where $g_r(x, y)$ −An arbitrary analytical function.

$Q_{i,j}$ −The constant coefficients determined from equation (1)

Substituting (4) in (1), we get,

$$b_0 \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} Q_{i,j} \frac{\partial^{2(i+j+2)}}{\partial x^{2(i+2)} \partial y^{2j}} g(x,y) \frac{t^{i+j+r}}{(i+j+r)!} +$$

$$+b_1 \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} Q_{i,j} \frac{\partial^{2(i+j+2)}}{\partial x^{2(i+1)} \partial y^{2(j+1)}} g(x,y) \frac{t^{i+j+r}}{(i+j+r)!}$$

$$+b_2 \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} Q_{i,j} \frac{\partial^{2(i+j+2)}}{\partial x^{2i} \partial y^{2(j+2)}} g(x,y) \frac{t^{i+j+r}}{(i+j+r)!} +$$

$$+ \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} Q_{i,j} \frac{\partial^{2(i+j)}}{\partial x^{2i} \partial y^{2j}} g(x,y) \frac{t^{i+j+r-2}}{(i+j+r-2)!} = 0$$

Replacing the index i in the first double sum with i-2, in the second i-with i-1,

j-with j-1 and in the third sum j-with j-2, and after combining all the sums into one, we get

$$\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} [b_0 Q_{i-2,j} + b_1 Q_{i-1,j-1} + b_2 Q_{i-1,j-1} + Q_{i,j}] \frac{\partial^{2(i+j)}}{\partial x^{2i} \partial y^{2j}} g(x,y) \frac{t^{i+j-2}}{(i+j-2)!} = 0$$

In order for this equality to hold for all values of the common factor, the expression in square brackets must be set to zero. As a result, the problem is reduced to solving the following numerical recurrence relation

$$Q_{i,j} = -(b_0 Q_{i-2,j} + b_1 Q_{i-1,j-1} + b_2 Q_{i,j-2}) \qquad (5)$$

Under initial conditions

$$Q_{i,j} = 0 \text{ By } i < 0 \text{or} j < 0; \quad Q_{0,0} = 1. \qquad (6)$$

Here are the first few coefficients $Q_{i,j}$:

$$Q_{0,0} = 1, \quad Q_{0,1} = 0, \quad Q_{0,2} = -b_2, \quad Q_{0,3} = 0, \cdots$$
$$Q_{1,0} = 0, \quad Q_{1,1} = -b_1, \quad Q_{1,2} = 0, \quad Q_{1,3} = 2b_1 b_2, \cdots$$
$$Q_{2,0} = -b_0, \quad Q_{2,1} = 0, \quad Q_{2,2} = 2b_0 b_2 + b_1^2, \quad Q_{2,3} = 0, \cdots$$
$$Q_{3,0} = 0, \quad Q_{3,1} = 2b_0 b_1, \quad Q_{3,2} = 0, \quad Q_{3,3} = -4b_0 b_1 b_2 - b_1(2b_0 b_2 + b_1^2), \cdots$$
$$\cdots$$

By the method of complete mathematical induction, we can show the validity of the expressions

$$Q_{2i,2j} = (-1)^{i+j} \sum_{s=0}^{i} \frac{(i+j)!}{s!(2i-2s)!(j-i+s)!} b_0^s b_1^{2(i-s)} b_2^{j-i+s},$$

$$Q_{2i+1,2j+1} = (-1)^{i+j+1} \sum_{s=0}^{i} \frac{(i+j+1)!}{s!(2i-2s+1)!(j-i+s)!} b_0^s b_1^{2(i-s)+1} b_2^{j-i+s}; \ (7)$$

In this case, the terms with negative powers at b_2 should be considered equal to zero.

Note that the sum of all the coefficients is$Q_{i,j}$, for which$i + j = 2k$,

$(-1)^k (b_0 + b_1 + b_2)$.

In the isotropic case, i.e., when the coefficients in equation (1) are equal $b_0 = b_2 = b^2$, $b_1 = 2b^2$, we get$Q_{i,j} = 0$при$i + j$odd and

$Q_{i,j} = (-1)^{\frac{i+j}{2}} \frac{(i+j)!}{i!j!} b^{i+j}byi + j$even-numbered.

By directly using the recurrent relation (5), taking into account (6) , we can make sure that the coefficients$Q_{i,j}$ with an odd sum of the indices i and j are zero.

Changing the order of summation, we present (4) as:

$$w = \sum_{i=0}^{\infty} \left(\sum_{j=0}^{i} Q_{i-j,j} \partial_x^{2(i-j)} \partial_y^{2j} (g) \right) \frac{t^{i+r}}{(i+r)!}.$$

Substituting (7) into this expression and collapsing the internal sum, we get the solution of equation (1) in the following operator form:

$$w = \sum_{i=0}^{\infty} (-1)^i \Delta_{xy}^{2i} (g) \frac{t^{2i+r}}{(2i+r)!}, \quad (8)$$

Where$\Delta_{xy}^{2i}() \equiv (b_0 \partial_x^4 + b_1 \partial_x^2 \partial_y^2 + b_2 \partial_y^4)^i ()$.

For function (4), the following differentiation formulas are true:

$$\frac{\partial}{\partial x} w_r(g) = w_r(\frac{\partial}{\partial x} g), \quad \frac{\partial}{\partial y} w_r(g) = w_r(\frac{\partial}{\partial y} g), \quad \frac{\partial}{\partial t} w_1(g) = w_0(g),$$

$$\frac{\partial}{\partial t} w_0(g) = \frac{\partial}{\partial t} g + \left[b_0 w_1(\frac{\partial^4}{\partial x^4} g) + b_1 w_1(\frac{\partial^4}{\partial x^2 \partial y^2} g) + b_2 w_1(\frac{\partial^4}{\partial y^4} g) \right]. \quad (9)$$

We find a solution to the Cauchy problem for equation (1) under the initial conditions

$$w\Big|_{t=0} = \phi_0(x,y), \qquad \frac{\partial w}{\partial t}\Big|_{t=0} = \phi_1(x,y) \quad (10)$$

We will look for a solution to the problem (1), (9) in the form

$$w = w_0(g_0) + w_1(g_1). \quad (11)$$

Substituting (11) into (10), using the differentiation formulas (9) and considering that$w_0(g)|_{t=0} = g$, $w_1(g)|_{t=0} = 0$, получим

$$g_0 = \phi_0, \quad g_1 = \phi_1.$$

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Therefore, the solution to problem (1), (10) is the function

$$w(x,y,t) = w_0(\phi_0) + w_1(\phi_1) \quad (12)$$

Let

$$\phi_0 = \xi_{\lambda,m,n}(x,y), \quad \phi_1 = 0, \quad (13)$$

Where

$$\xi_{1,m,n}(x,y) = \sin a_m x \sin b_n y; \quad \xi_{2,m,n}(x,y) = \cos a_m x \sin b_n y;$$

$$\xi_{3,m,n}(x,y) = \sin a_m x \cos b_n y; \xi_{4,m,n}(x,y) = \cos a_m x \cos b_n y. \quad (14)$$

Substituting (14) in (13) and taking into account (4), we get

$$w_0^{\lambda,m,n} =$$

$$= \xi_{\lambda,m,n} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} (-1)^{i+j} Q_{i,j} a_m^{2i} a_n^{2j} \frac{t^{(i+j)}}{(i+j)!} =$$

$$= \xi_{\lambda,m,n} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} (-1)^{i+j} Q_{i,j} a_m^{2i} a_n^{2j} \frac{t^{(i+j)}}{(i+j)!} =$$

$$= \xi_{\lambda,m,n} \sum_{k=0}^{\infty} (-1)^k (b_0 a_m^4 + b_1 a_m^2 b_n^2 + b_2 b_n^4)^k \frac{t^{2k}}{(2k)!} = \xi_{\lambda,m,n} \cos\sqrt{\delta} t,$$

(15)

Where

$$\delta = b_0 a_m^4 + b_1 a_m^2 b_n^2 + b_2 b_n^4 \quad (16)$$

The coefficients property is used here$Q_{i,j} = 0$by$i + j = 2k + 1$, and also the ratio

$$\sum_{i=0}^{2k} Q_{i,2k-i} a_m^{2i} b_n^{2(2k-i)} = (-1)^k \delta^k.$$

In the case of an isotropic plate, we obtain

$$w_0^{\lambda,m,n} = \xi_{\lambda,m,n} \cos[b(a_m^2 + b_n^2)t] \quad (17)$$

The function (15) satisfies equation (1) and the initial conditions (13).

If the initial conditions are given as

$$\phi_0 = 0, \quad \phi_1 = \xi_{\lambda,m,n}(x,y), \quad (18)$$

Then the solution to the Cauchy problem will have the form

$$w_1^{\lambda,m,n} = \xi_{\lambda,m,n} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} (-1)^{i+j} Q_{i,j} a_m^{2i} a_n^{2j} \frac{t^{(i+j+1)}}{(i+j+1)!} = \xi_{\lambda,m,n} \frac{1}{\sqrt{\delta}} \sin\sqrt{\delta} t.$$

(19)

In the case of an isotropic plate, the solution (19) will take the form

$$w_1^{\lambda,m,n} = \xi_{\lambda,m,n} \frac{1}{b(a_m^2 + b_n^2)} \sin[b(a_m^2 + b_n^2)t] \quad (20)$$

The function (19) satisfies equation (1) and the initial conditions (18). We

Proceed to the solution of the inhomogeneous equation

$$L(w) \equiv \left(b_0 \frac{\partial^4}{\partial x^4} + b_1 \frac{\partial^4}{\partial x^2 \partial y^2} + b_2 \frac{\partial^4}{\partial y^4} + \frac{\partial^2}{\partial t^2}\right) w = f(x,y,t) \quad (21)$$

In accordance with the recurrent operator method [6, 7], the solution of equation (1) is

$$w_3(f) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} Q_{i,j} \frac{\partial^{2(i+j)}}{\partial x^{2i} \partial y^{2j}} \frac{\partial^{-(i+j+2)}}{\partial t^{-(i+j+2)}}(f).$$

(22)

$Q_{i,j}$ -it is taken by the formula (5), $\frac{\partial^{-s}}{\partial t^{-s}}(f) = \int_0^t \cdots \int_0^t f(x,y,t)\, dt^s$.

The general solution of equation (1) is the sum of solutions (4) and (21). Assuming

F (x, y, t)=ϕ(x, y) ψ (t), we represent (22)

$$\begin{aligned} w_3(f) = -(&Q_{0,0} f^{-2} + Q_{0,1} f_{y^2}^{-3} + Q_{0,2} f_{y^4}^{-4} + Q_{0,3} f_{y^6}^{-5} + \cdots \\ &+ Q_{1,0} f_{x^2}^{-3} + Q_{1,1} f_{x^2 y^2}^{-4} + Q_{1,2} f_{x^2 y^4}^{-5} + \cdots \\ &+ Q_{2,0} f_{x^4}^{-4} + Q_{2,1} f_{x^4 y^2}^{-5} + \cdots \\ &+ Q_{3,0} f_{x^6}^{-5} + \cdots) = \end{aligned}$$

$$\begin{aligned} = \Big[&Q_{0,0} \phi \psi^{-2} + \underbrace{(Q_{1,0} \partial_{x^2} + Q_{0,1} \partial_{y^2})}_{=0} (\phi)(\psi)^{-3} + \\ &+ (Q_{2,0} \partial_{x^4} + Q_{1,1} \partial_{x^2 y^2} + Q_{0,2} \partial_{y^4})(\phi)(\psi)^{-4} + \\ &\underbrace{(Q_{3,0} \partial_{x^6} + Q_{2,1} \partial_{x^4 y^2} + Q_{1,2} \partial_{x^2 y^4} + Q_{0,3} \partial_{y^6})}_{=0} (\phi)(\psi)^{-5} + \\ &+ (Q_{4,0} \partial_{x^8} + Q_{3,1} \partial_{x^6 y^2} + Q_{2,2} \partial_{x^4 y^4} + \\ &+ Q_{1,3} \partial_{x^2 y^6} + Q_{0,4} \partial_{y^8})(\phi)(\psi)^{-6} + \cdots \Big] \end{aligned}$$

Adding expressions $Q_{i,j}$ from (5), we get

$$w_3(f) = w_3(\phi(x,y)\psi(t)) = \sum_{k=0}^{\infty} (-1)^k \Delta_{xy}^{2k}(\phi) \int_0^t \cdots \int_0^t \psi(t) dt^{2k+2}. \quad (23)$$

<u>Example.</u>We solve the problem of forced vibrations of a rectangular, orthotropic, hinged plate. The problem is reduced to solving equation (21) under the following boundary and initial conditions

$$w\Big|_{\substack{x=0,a \\ y=0,b}} = \frac{\partial^2 w}{\partial x^2}\Big|_{x=0,a} = \frac{\partial^2 w}{\partial y^2}\Big|_{y=0,b} = 0,\ w\Big|_{t=0} = 0,\ \frac{\partial}{\partial t} w\Big|_{t=0} = 0.$$

$f = \sin\frac{\pi}{20}x\sin\frac{\pi}{20}\cos\omega t$, $b_0 = 0{,}735 \cdot 10^{-5}\frac{см^2}{кг}$, $b_1 = 1{,}2436 \cdot 10^{-5}\frac{см^2}{кг}$, $b_2 = 0{,}508 \cdot 10^{-5}\frac{см^2}{кг}$, $\omega = 20c^{-1}$(the material is fiberglass CAST-In) [3,4].

Let's solve the test problem of mechanical vibrations for a rectangular ($0 \le x \le a,\ 0 \le y \le b$) plates in the case of equation (1) with the right-hand side

$$f = \sin\frac{\pi x}{a}\sin\frac{\pi y}{b}\cos\omega t, \qquad (24)$$

under the following conditions

$$w\Big|_{\substack{x=0,a\\ y=0,b}} = \frac{\partial^2 w}{\partial x^2}\Big|_{x=0,a} = \frac{\partial^2 w}{\partial y^2}\Big|_{y=0,b} = 0, w\Big|_{t=0}0,\ \frac{\partial}{\partial t}w\Big|_{t=0} = 0. \quad (25)$$

We assume the following initial data a=b=20,. The rest of the data is taken from the previous example [8, 9]. Substituting (24) in (21), we obtain solutions to equation (1) in the form

$$w_3(f) = \sin\frac{\pi}{a}x\sin\frac{\pi}{b}y\sum_{k=0}^{\infty} -\left[\left(b_0\left(\frac{\pi}{a}\right)^4 + b_1\left(\frac{\pi}{a}\right)^2\left(\frac{\pi}{b}\right)^2 + b_2\left(\frac{\pi}{b}\right)^4\right)^k\left(\frac{1}{\omega^2}\right)^{k+1} \times \times\left(\cos\omega t + \sum_{i=0}^{k}(-1)^{i+1}\frac{\omega^{2i}t^{2i}}{(2i)!}\right)\right] \qquad (26)$$

Solutions (26) satisfy (1) and initial boundary conditions (25). According to the formula (26), graphs are plotted (Fig. 1) of the oscillations of the central point of the plate from the impact (24). Figure 2 shows graphs of changes in the shape of the oscillation over time (t=0.2 s, 0.4 s, 0.5 s) in the cross section x=10 cm.

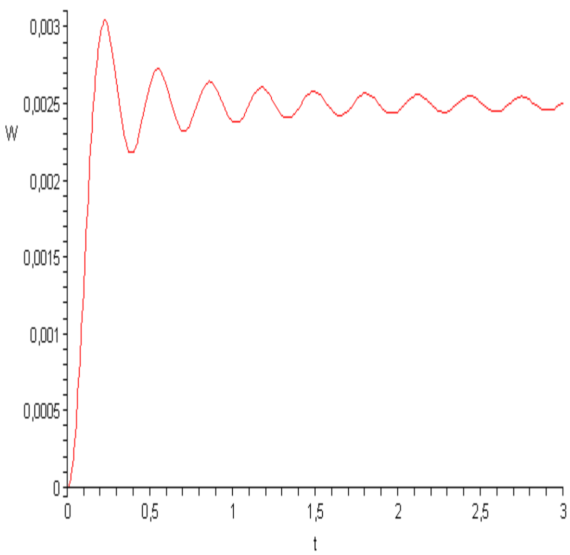

Fig 1

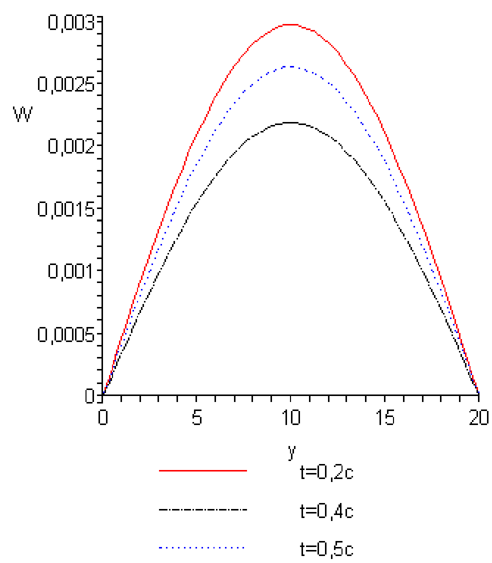

Fig 2

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