

DOI: [10.5958/2249-7137.2021.01135.6](https://doi.org/10.5958/2249-7137.2021.01135.6)

FIND THE LENGTHS OF TWO TRAJECTORIES THAT DIVIDE AN ARBITRARY TRIANGLE INTO THREE EQUAL ANGLES

Norkoziev Navruzбек Noralioglu*

*Mathematics Teacher,
56th comprehensive school of Sherabaddistrict of Surkhandarya region,
UZBEKISTAN

Email id: navruzbeknorqoziyev@mail.ru

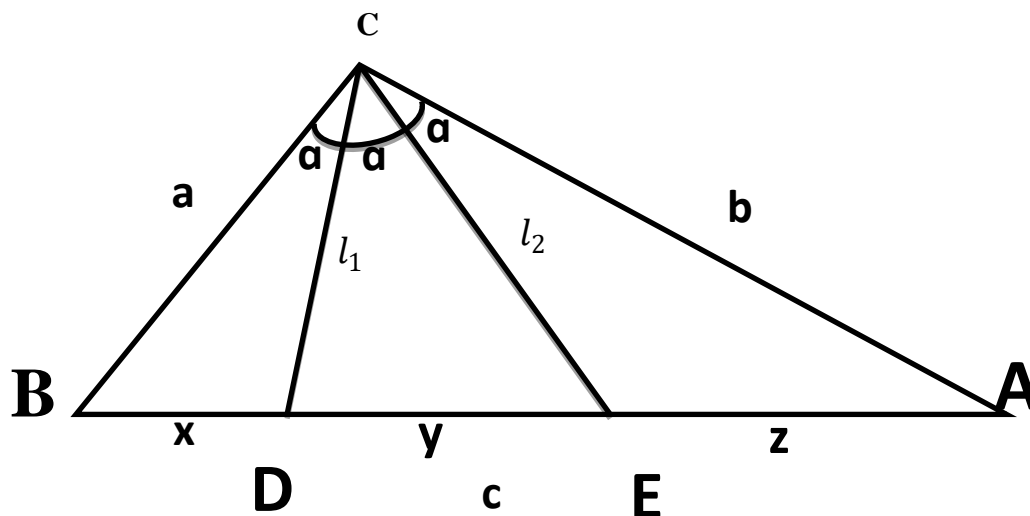
ABSTRACT

In this paper, the angle of a triangle can be divided into 3 equal parts, and it is given to find 2 tresektrissa of different lengths coming out of one angle.

KEYWORDS: Triangle, Angle, Triangle Surface, Bisector Ratio, $\sin a \cos 2 a$, $\sin^2 a$, Triangle Ratio And Sides Ratio.

INTRODUCTION

If we are given two sides of an arbitrary triangle ABC and one angle between them. Find two tresektrissa l_1 and l_2 that divide one of the angles of the triangle into three equal parts.



The sides of a triangle $BC=a$, $AC= b$, $AB=c$ sides and $\angle BCA=3\alpha$ let's say. Here are two bicycles AB side x , y and z to lengths. $AB=c= x+y+z$ get. Triangle BCE Let us give this formula when the angle of a triangle is equal to two.

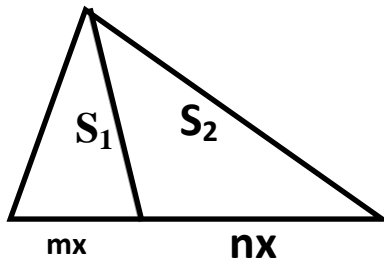
$$\frac{a}{x} = \frac{l_2}{y}$$

Triangle DAC and for these formulas, l_1 va l_2 Let's simplify them.

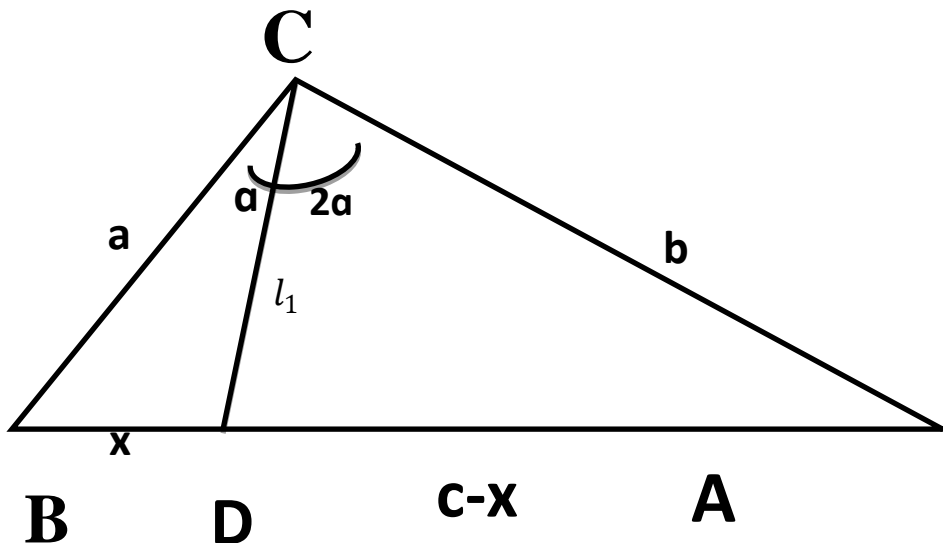
$$\frac{l_1}{y} = \frac{b}{z}$$

$$l_1 = b \frac{y}{z} l_2 = a \frac{y}{x}$$

Now we find that x , y , and z are the sides of a and b and the 3α between them. To do this, we need this formula. The side of a triangle has the same proportion as the opposite side of the straight line from the end of the triangle..



$\frac{S_1}{S_2} = \frac{m}{n}$ So based on this formula we get the ratio of the triangles BCD and CAD .



$$\frac{S_{BDC}}{S_{CDA}} = \frac{x}{c-x}$$

Now we write this on the surfaces of a triangle, come to the following formula, and simplify it by finding the x by shortening the denominator.

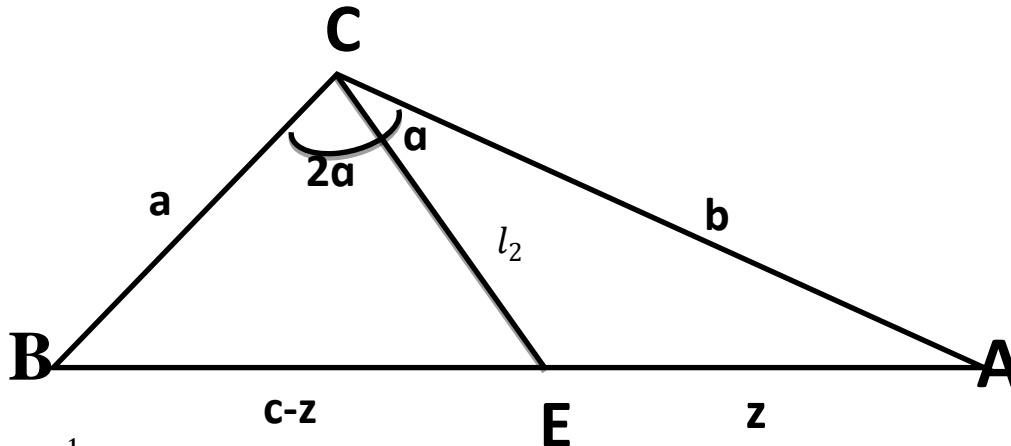
$$\frac{\frac{1}{2} a l_1 \sin \alpha}{\frac{1}{2} l_1 b \sin 2\alpha} = \frac{x}{c-x} \frac{a \sin \alpha}{2 b \sin \alpha \cos \alpha} = \frac{x}{c-x} \frac{a}{2 b \cos \alpha} = \frac{x}{c-x}$$

$$a(c-x) = 2b \times \cos a \quad ac - ax = 2bx \cos a$$

$$ac = ax + 2bx \cos a \quad x(a + 2b \cos a) = ac$$

$$x = \frac{ac}{a + 2b \cos a}$$

So we found x and now we can find z in the same way.



$$\frac{S_{CEA}}{S_{BCE}} = \frac{z}{c-z} \frac{\frac{1}{2}bl_2 \sin a}{\frac{1}{2}al_2 \sin 2a} = \frac{z}{c-z} \frac{b}{2a \cos a} = \frac{z}{c-z}$$

$$b(c-z) = 2az \cos a \quad bc - bz = 2az \cos a$$

$$z = \frac{bc}{b + 2a \cos a}$$

So we found z, now we just have to find y. For this $y = c - (x+z)$ we use.

$$y = c - x - z$$

$$y = c - \frac{ac}{a + 2b \cos a} - \frac{bc}{b + 2a \cos a} \quad y = c \left(1 - \frac{a}{a + 2b \cos a} - \frac{b}{b + 2a \cos a} \right)$$

$$y = c \left(\frac{(a + 2b \cos a)(b + 2a \cos a) - a(b + 2a \cos a) - b(a + 2b \cos a)}{(a + 2b \cos a)(b + 2a \cos a)} \right)$$

Now we can just simplify the image by opening the parentheses and narrowing down the similar terms.

$$y = c \left(\frac{ab + 2a^2 \cos a + 2b^2 \cos a + 4ab \cos^2 a - ab - 2a^2 \cos a - ab - 2b^2 \cos a}{(a + 2b \cos a)(b + 2a \cos a)} \right)$$

$$y = c \left(\frac{4ab \cos^2 a - ab}{(a + 2b \cos a)(b + 2a \cos a)} \right) \quad y = \frac{abc(4 \cos^2 a - 1)}{(a + 2b \cos a)(b + 2a \cos a)}$$

So we found x, y, and z by the sides a, b, and the angle cosa

l_1 and l_2 by shortening similar terms by placing them in place to find them

l_1 and l_2 we can find.

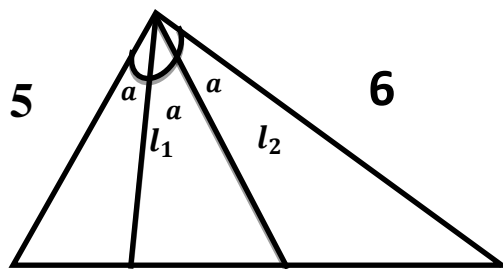
$$l_1 = b \frac{y}{z} \quad l_2 = a \frac{y}{x}$$

$$l_1 = b \frac{\frac{abc(4 \cos^2 a - 1)}{(a+2bc \cos a)(b+2a \cos a)}}{\frac{bc}{b+2a \cos a}} l_1 = \frac{ab(4 \cos^2 a - 1)}{a+2bc \cos a}$$

$$l_2 = a \frac{\frac{abc(4 \cos^2 a - 1)}{(a+2bc \cos a)(b+2a \cos a)}}{\frac{ac}{a+2bc \cos a}} l_2 = \frac{ab(4 \cos^2 a - 1)}{b+2a \cos a}$$

So we have found l_1 and l_2 , now that we are working on examples of these formulas and checking that the faces of the three triangles separated by these tresektrissa must be equal to the faces of the common triangles.

Example 1: If the sides of a triangle are 5cm, 6cm and the angle between them is given by 10° , and find the angular tresektrissa that divide this angle into three equal parts.



$$a=5 \quad b=6 \quad 3\alpha = 10 \text{ equal .}$$

The face of a triangle

$$S = \frac{1}{2} \cdot 5 \cdot 6 \sin 10^\circ = 15 \sin 10^\circ$$

$$l_1 = \frac{ab(4 \cos^2 a - 1)}{a+2bc \cos a} = \frac{5 \cdot 6(4 \cos^2 \frac{10}{3} - 1)}{5+12 \cos \frac{10}{3}} = \frac{30(4 \cos^2 \frac{10}{3} - 1)}{5+12 \cos \frac{10}{3}}$$

$$l_2 = \frac{ab(4 \cos^2 a - 1)}{b+2a \cos a} = \frac{30(4 \cos^2 \frac{10}{3} - 1)}{6+10 \cos \frac{10}{3}}$$

Now let's use these tresektrissa to check that the sum of the three triangles is equal to the total area of the triangle.

$$15 \sin 10^\circ = S_1 + S_2 + S_3 \quad S_1 = \frac{1}{2} a l_1 \sin \frac{10}{3} \quad S_2 = \frac{1}{2} l_1 l_2 \sin \frac{10}{3} \quad S_3 = \frac{1}{2} b l_2 \sin \frac{10}{3}$$

$$S = \frac{1}{2} \cdot 5 \cdot \frac{30(4 \cos^2 \frac{10}{3} - 1)}{5+12 \cos \frac{10}{3}} \sin \frac{10}{3} + \frac{1}{2} \cdot \frac{30(4 \cos^2 \frac{10}{3} - 1)}{5+12 \cos \frac{10}{3}} \cdot \frac{30(4 \cos^2 \frac{10}{3} - 1)}{6+10 \cos \frac{10}{3}} \sin \frac{10}{3} + \frac{1}{2} \cdot 6 \cdot \frac{30(4 \cos^2 \frac{10}{3} - 1)}{6+10 \cos \frac{10}{3}} \sin \frac{10}{3}$$

Andy $\frac{1}{2} \cdot 30 \cdot \sin \frac{10}{3} \cdot (4 \cos^2 \frac{10}{3} - 1)$ out of parentheses.

$$S = 15 \cdot \sin \frac{10}{3} \cdot (4 \cos^2 \frac{10}{3} - 1) \left(\frac{5}{5+12 \cos \frac{10}{3}} + \frac{30(4 \cos^2 \frac{10}{3} - 1)}{(5+12 \cos \frac{10}{3})(6+10 \cos \frac{10}{3})} + \frac{6}{6+10 \cos \frac{10}{3}} \right)$$

$$S = 15 \cdot \sin \frac{10}{3} \cdot (4 \cos^2 \frac{10}{3} - 1) \left(\frac{5(6+10 \cos \frac{10}{3}) + 30(4 \cos^2 \frac{10}{3} - 1) + 6(5+12 \cos \frac{10}{3})}{(5+12 \cos \frac{10}{3})(6+10 \cos \frac{10}{3})} \right)$$

$$S=15 \cdot \sin \frac{10}{3} \cdot (4 \cos^2 \frac{10}{3} - 1) \left(\frac{30+50\cos \frac{10}{3}+120\cos^2 \frac{10}{3}-30+30+72\cos \frac{10}{3}}{30+50\cos \frac{10}{3}+72\cos \frac{10}{3}+120\cos^2 \frac{10}{3}} \right)$$

$$S=15 \cdot \sin \frac{10}{3} \cdot (4 \cos^2 \frac{10}{3} - 1) \left(\frac{120\cos^2 \frac{10}{3}+122\cos \frac{10}{3}+30}{120\cos^2 \frac{10}{3}+122\cos \frac{10}{3}+30} \right) = 15 \cdot \sin \frac{10}{3} \cdot (4 \cos^2 \frac{10}{3} - 1) \cdot 1$$

Now let's add an extra formula here.

$$\sin 3a = \sin (2a + a) = \sin 2a \cdot \cos a + \sin a \cdot \cos 2a = 2\sin a \cdot \cos a \cdot \cos a + \sin a \cdot (2\cos^2 a - 1) =$$

$$= 2\sin a \cdot \cos^2 a + 2\sin a \cdot \cos^2 a - \sin a = 4\sin a \cdot \cos^2 a - \sin a =$$

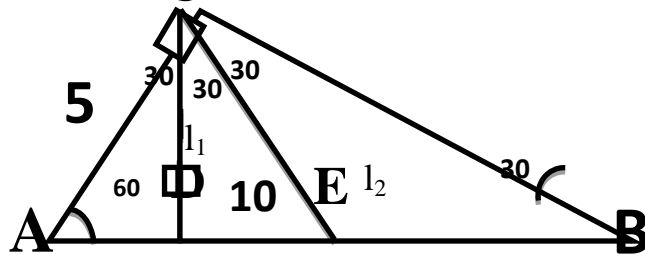
$$= \sin a (4 \cos^2 a - 1)$$

$$S \sin \frac{10}{3} \cdot (4 \cos^2 \frac{10}{3} - 1) = \sin 3 \cdot \frac{10}{3} = \sin 10^\circ$$

For example -2 :If the catheter of a right triangle AC=5 smandhypotenuse

AB=10 smis equal to, C find the tresektrissa that divide the angle into three equal angles?

$$\text{Ergo } BC = \sqrt{10^2 - 5^2} = \sqrt{75} = 5\sqrt{3}$$



$$\sin B = \frac{5}{10} \sin B = \frac{1}{2} \angle B = 30^\circ$$

$$\angle A = 60^\circ \text{ equally}$$

Ergo $\angle A = \angle E$ from equality $AC = CE = l_2 = 5$ it turns out that,

$\sin B = \frac{l_1}{BC}$ which is derived from the triangular CDB and is calculated by putting it in place l_1 let's

find $l_1 = BC \cdot \sin B = 5\sqrt{3} \cdot \sin 30^\circ = \frac{5\sqrt{3}}{2}$ equality arises .Now created in the beginning l_1 and

l_2 let's recalculate and check for equality. Here $\alpha = 30^\circ$ equal.

$$l_1 = \frac{ab(4 \cos^2 a - 1)}{a + 2b \cos a} = \frac{5 \cdot 5\sqrt{3}(4 \cos^2 30^\circ - 1)}{5 + 2 \cdot 5\sqrt{3} \cos 30^\circ} = \frac{25\sqrt{3}(4 \cdot \frac{3}{4} - 1)}{5 + 10\sqrt{3} \cdot \frac{\sqrt{3}}{2}} = \frac{50\sqrt{3}}{20} = \frac{5\sqrt{3}}{2}$$

$$l_2 = \frac{ab(4 \cos^2 a - 1)}{b + 2a \cos a} = \frac{5 \cdot 5\sqrt{3}(4 \cos^2 30^\circ - 1)}{5\sqrt{3} + 2 \cdot 5 \cos 30^\circ} = \frac{25\sqrt{3}(4 \cdot \frac{3}{4} - 1)}{5\sqrt{3} + 5\sqrt{3}} = \frac{50\sqrt{3}}{10\sqrt{3}} = 5$$

It follows that the formulas we have created are correct.

REFERENCES

1. Geometry book for secondary school grades 8 Author: A.A. Rahimqoriyev, M.A. To'xtahodjoyeva. Toshkent -2019.
2. Geometry book for secondary school grades 9 Author: B Haydarov, E Sariqov, A Qo'shqorov. Toshkent -2019. 147-paje.